

$$H = \frac{1}{4!} \sum_{ijkl=1}^N J_{ijkl} x_i x_j x_k x_l$$

$$J_{ijkl}; \quad i=1, \dots, N \quad N \rightarrow \infty$$

$$\overline{J_{ijkl}} = 0$$

$$\{x_i, x_j\} = \delta_{ij}$$

$$\overline{J_{ijkl}^2} = \frac{3! J^2}{N^3}$$

$$\langle J_{ijkl} J_{i'j'k'l'} \rangle = (\delta_{ii'} \delta_{jj'} \delta_{kk'} \delta_{ll'} - \dots)$$

$$H = \sum_{i_j < k < l} J_{ijkl} x_i x_j x_k x_l$$

$$\frac{N(N-1)(N-2)(N-3)}{12}; \quad \overline{J_{ijkl}} \quad N^4$$

$$2^{N/2} \times 2^{N/2}$$

$$x_1 = \{ \sigma_x \otimes \mathbb{1} \otimes \mathbb{1} \dots \otimes \mathbb{1} \}$$

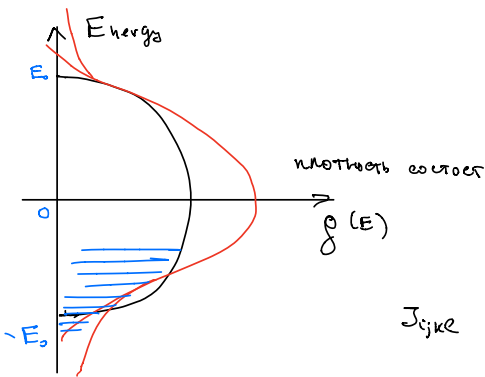
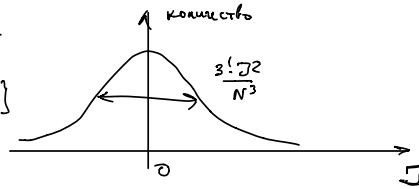
$$N = 2 \text{ et } 4 \text{ et } 6$$

$$x_2 = \{ \sigma_y \otimes \mathbb{1} \otimes \mathbb{1} \dots \otimes \mathbb{1} \}$$

$$x_3 = \{ \sigma_z \otimes \sigma_x \otimes \mathbb{1} \dots \otimes \mathbb{1} \}$$

$$\sigma_z \otimes \sigma_z \dots \otimes \sigma_z \otimes \sigma_x \otimes \dots \otimes \mathbb{1}$$

$$\{ J_{1234}, J_{1256}, \dots, J_{23710} \}$$



$$H = \sum J_{ijkl} c_i^\dagger c_j^\dagger c_k c_l \quad \underline{170}$$

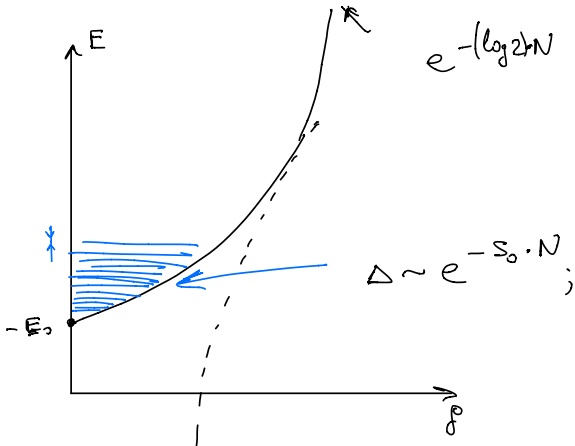
$$\{c_i^\dagger, c_j\} = \delta_{ij}$$

$$\{c_i, c_j\} = \{c_i^\dagger, c_j^\dagger\} = 0$$

$$J_{ijkl} = \int d^3r d^3r' \psi_i^*(r) \psi_j^*(r') V(r-r') \psi_k(r) \psi_l(r')$$

$$e^{-(\log 2)N}$$

$$2^N \sim e^{(\log 2) \cdot N}$$



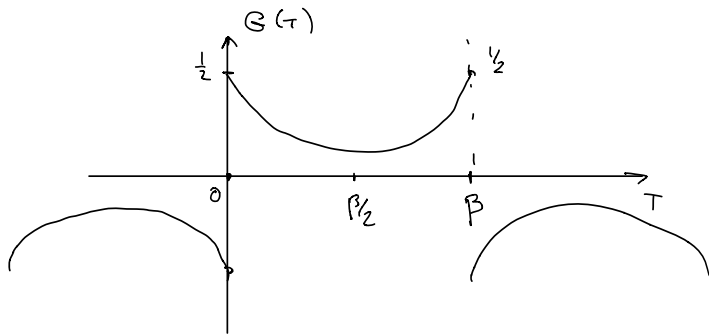
$$s_0 = 0.23$$

$$\boxed{N \rightarrow \infty}$$

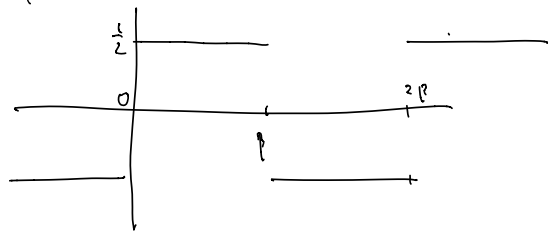
$$G(\tau) = \frac{1}{N} \sum_{i=1}^N \langle T \chi_i(\tau) \chi_i(0) \rangle_{\beta} = \begin{cases} \frac{1}{2} \text{Tr} (e^{\tau H} x e^{-\tau H} x e^{-\beta H}); & \tau > 0 \\ & \tau < 0 \end{cases}$$

$$\tau = 0 \quad x^2 = \frac{1}{2} \quad \{ \chi_i, \chi_j \} = \delta_{ij} \quad G(0^+) = \frac{1}{2} \quad \beta/2$$

$$G(\beta^-) = \frac{1}{2} \quad \beta = 1/T$$



$$H=0 \quad G_0(\tau) = \frac{1}{2} \text{sgn} \tau$$



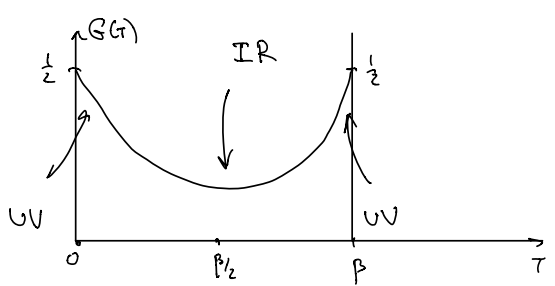
$$G(\tau) = \frac{G(\tau)}{G_0(\tau)} = \frac{1}{2} + \dots$$

Diagram showing the expansion of the ratio $G(\tau)/G_0(\tau)$ as a series of terms. The first term is $1/2$. Subsequent terms are represented by circles on a horizontal line, with dashed lines indicating interactions between them. A small diagram shows a cross with axes x_i, x_j, x_k, x_l and a label J_{ijke} .

$$\frac{1}{J_{ijke}^2} = \frac{1}{N^3}$$

$$\tau_{12} = \tau_1 - \tau_2 = \tau_1 - \tau_3 + \tau_3 - \tau_4 + \tau_4 - \tau_2$$

$$G(\tau_{12}) = G_0(\tau_{12}) + \int_0^{\beta} d\tau_3 d\tau_4 G_0(\tau_{13}) G^3(\tau_{34}) G(\tau_{42})$$



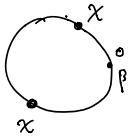
$$T \rightarrow 0 \quad \beta \rightarrow \infty$$

$$\Sigma(\tau) = \int^2 G^3(\tau)$$

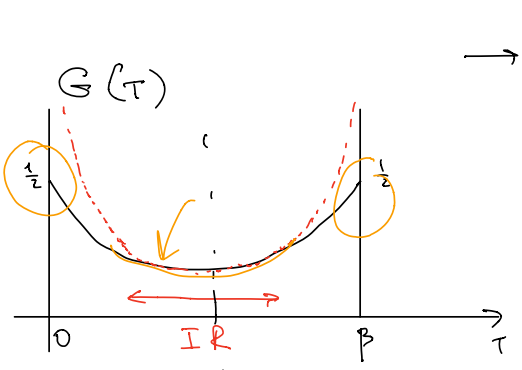
$$G(\omega_n) = \frac{1}{i\omega_n - \Sigma(\omega_n)}$$

$$G(\omega_n) = \int_0^{\beta} d\tau G(\tau) e^{i\omega_n \tau}$$

$$\omega_n = \frac{2\pi}{\beta} (n + \frac{1}{2})$$



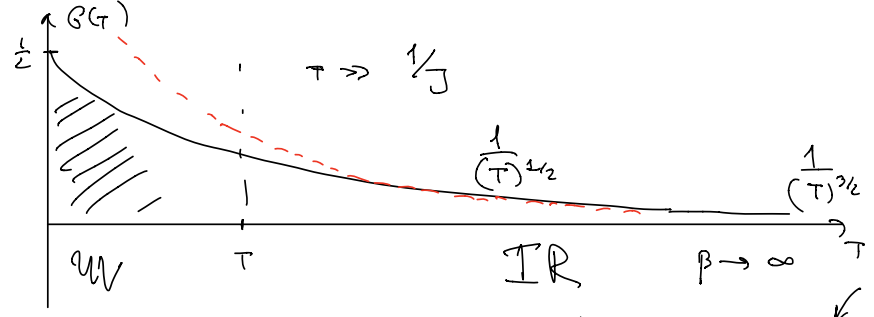
$$G(\tau) = \frac{1}{\left(\sin \frac{\pi \tau}{\beta}\right)^{2\Delta}} \quad \Delta = 1/4 \quad (\beta J) \rightarrow \infty$$



$$\rightarrow \frac{1}{|\tau|^{2\Delta}} = \frac{1}{\tau^{1/2}}$$

$$\beta J \gg 1 \quad (N \rightarrow \infty \gg \beta J)$$

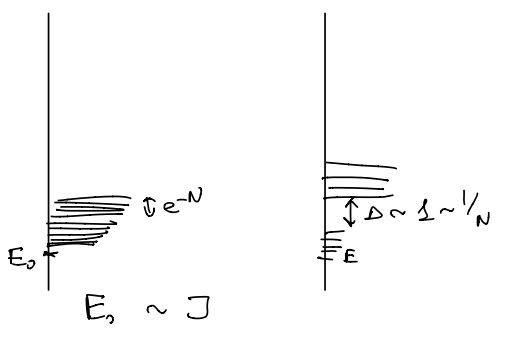
$$\tau \rightarrow 0 \quad \beta \rightarrow \infty$$



$$\underline{J = 1}$$

$$G(\tau) = \frac{1}{\left(\sin \frac{\pi \tau}{\beta}\right)^{1/2}} \left(1 + \frac{1}{(\beta J)} f(\tau) + \frac{1}{(\beta J)^2} f(\tau)^2 \right)$$

$$\sum_{n,m} \langle n | e^{\tau H} x e^{-\tau H} | m \rangle \langle m | x e^{-\beta H} | n \rangle = \sum_{n,m} |\langle n | x | m \rangle|^2 e^{-\beta E_n + \tau(E_m - E_n)}$$



$$\downarrow e^{-\Delta \tau}$$

$$\downarrow$$

$$\Delta \sim e^{-N}$$

$$F = -\log Z$$

$$\langle F \rangle_J = \langle \log Z \rangle_J = \lim_{n \rightarrow 0} \left\langle \frac{Z^n - 1}{n} \right\rangle_J \quad n = \mathbb{Z}$$

$$Z = \int D\chi_i e^{\int_0^T \chi_i (\partial_\tau) \chi_i - \int J_{ijke} \chi_i(\tau) \chi_j(\tau) \chi_k(\tau) \chi_l(\tau) e^G}$$

$$Z^n = \int D\chi_i^a e^{\int_0^T \chi_i^a \partial_\tau \chi_i^a - \int J_{ijke} \chi_i^a \chi_j^a \chi_k^a \chi_l^a} \quad a=1, \dots, n$$

$$\int dJ_{ijke} e^{-\frac{J_{ijke}^2}{2 \left(\frac{3! J^2}{N^3}\right)}} Z^n =$$

$$= \int D\chi_i^a e^{\int_0^T \chi_i^a \partial_\tau \chi_i^a - \frac{J^2}{N^3} \int d\tau d\tau' \chi_i^a(\tau) \dots \chi_l^a(\tau) \chi_i^b(\tau') \dots \chi_l^b(\tau')}$$

$$= \int D\chi_i^a e^{\int_0^T \chi_i^a \partial_\tau \chi_i^a - \frac{J^2}{N^3} \int d\tau d\tau' (\chi_i^a(\tau) \chi_i^b(\tau'))^4}$$

$$\int DG_{ab} \delta(G_{ab}(\tau, \tau') - \chi_i^a(\tau) \chi_i^b(\tau')) = \mathbb{1}$$

$$= \int DG_{ab} D\chi_i^a e^{\int d\tau \chi_i^a \partial_\tau \chi_i^a - \frac{J^2}{N^3} \int d\tau d\tau' (G_{ab}(\tau, \tau'))^4 \delta(G_{ab} - \chi_i^a \chi_i^b)}$$

=

$$\int DG D\Sigma e^{\underbrace{N \left[\log Pf(\Sigma) - \Sigma \right]}_{S[G, \Sigma]} + \int G(\tau, \tau') \Sigma(\tau, \tau') + \frac{J^2}{4} G^4}$$

G, Σ - action

$S[G, \Sigma]$

$$\frac{\delta S}{\delta G(\tau, \tau')} = 0 \quad \frac{\delta S}{\delta \Sigma} = 0$$



$$\Sigma = J^2 G^3(\tau)$$



G_*, Σ_*
↑

$$G(\omega) = \frac{f}{i\omega - \Sigma(\omega)}$$

$f(\tau)$ parameters,

$$G(\tau, \tau') \rightarrow (f(\tau) f(\tau'))^\Delta G(f(\tau), f(\tau'))$$

$\Delta = 1/4$

$$\Sigma(\tau, \tau') \rightarrow (f(\tau) f(\tau'))^{1-\Delta} \Sigma(f(\tau), f(\tau'))$$

$$S[G, \Sigma] \rightarrow S[G, \Sigma]$$

$$\partial_T - \Sigma \rightarrow \sigma_{(T, T')} - \Sigma_{(T, T')} = \tilde{\Sigma}_{(T, T')}$$

$$\underline{\sigma_{(T, T')} = \delta'(T - T')}$$

$$S[G, \tilde{\Sigma}] = \underbrace{\log Pf(\tilde{\Sigma}) - \int (G \tilde{\Sigma} + \frac{J^2}{4} G^4)}_{S_{\text{conf}}(G, \tilde{\Sigma})} + \underbrace{\int \sigma_{(T, T')} G_{(T, T')} dT dT'}_{\text{bosnyuyume}}$$

$$G_* \sim \frac{1}{|T - T'|^{1/2}} \rightarrow G_* = \frac{(f(T) f(T'))^{1/4}}{|f(T) - f(T')|^{1/2}}$$

$$\int_0^\beta d\tau \left\{ \tan \frac{f(\tau)}{2}, \tau \right\}$$

$$\{ \phi(T), \tau \} = \frac{\phi^{(0)}}{\phi'} - \frac{3}{2} \left(\frac{\phi''}{\phi'} \right)^2$$

$$S[G, \Sigma] = \log Pf(\partial_T - \Sigma) - \int_0^\beta (G \Sigma + \frac{J^2}{4} G^4)$$

$$\frac{\delta S}{\delta G} = \frac{\delta S}{\delta \Sigma} = 0$$

G_*, Σ_*

$$G = G_* + g; \quad \Sigma = \Sigma_* + \sigma$$

$$S[G, \Sigma] = S[G_*, \Sigma_*] + \int \sigma_{(T_2, T_2)} \Pi_{(T_2, T_1, T_3, T_4)} \sigma_{(T_3, T_4)} + \int \sigma g + \int g(\dots) g$$

$$= S[G_*, \Sigma_*] + \int g_{(T_2, T_2)} (K_{(T_2, T_1, T_3, T_4)} - 1) g_{(T_3, T_4)} dT_2 \dots dT_4$$

$$K_{(T_2, T_2; T_3, T_4)} = \frac{\tau_2 \tau_3}{\tau_2 \tau_4} = 3J^2 G_*(\tau_{23}) G_*(\tau_{24}) G_*^2(\tau_{34})$$

$$\int K_{(T_2, T_2; T_3, T_4)} \Phi_h(T_3, T_4) dT_3 dT_4 = K_h \Phi_h(T_2, T_2)$$

$$K_c = 3J^2 G_c(\tau_{23}) G_c(\tau_{24}) G_c^2(\tau_{34}) \quad \underline{G_c(\tau)} = \frac{1}{(s_{1/2} \frac{\pi \tau}{\beta})^{1/2}}$$

$$Z = \int \frac{D\phi}{Df(G^4)} e^{\int_0^\beta d\tau \{ e^{i\phi}, \tau \}} \quad \downarrow \quad \left(\frac{\phi'''}{\phi'} + \left(\frac{\phi''}{\phi'} \right)^2 \right) \quad \text{Witten, Stanford}$$

$\phi(2\pi) = \phi(0) + 2\pi$

$$\langle G(\tau_1, \tau_2) \rangle = \int Df \frac{(f(\tau_1) f(\tau_2))^\Delta}{|f(\tau_1) - f(\tau_2)|^{2\Delta}} e^{\gamma \int d\tau \left\{ \tan \frac{f(\tau)}{2}, \tau \right\}} = \dots$$

$$K_c * \Phi_{h=2} = 1 \cdot \Phi_{h=2} \quad \int g \binom{k-1}{q} g^{\leftarrow \Phi_{h=2}} = \int$$

\downarrow
 $1 - \frac{c}{\beta}$
