$$H = \frac{1}{4!}\sum_{ijkl=1}^{N} J_{ijkl}\,\chi_i \chi_j \chi_k \chi_l \qquad J_{ijkl}\ ; \quad i = 1,\dots, N \qquad N \to \infty$$

$$\overline{J_{ijkl}} = 0 \qquad \{\chi_i, \chi_j\} = \delta_{ij}$$

$$\overline{J^2_{ijkl}} = \frac{3!\,J^2}{N^3}$$

$$\langle J_{ijkl} J_{i'j'k'l'} \rangle = \left(\delta_{ii'}\delta_{jj'}\delta_{kk'}\delta_{ll'} - \dots \quad . \right)$$

$$H = \sum_{i<j<k<l}^{N} J_{ijkl}\,\chi_i \chi_j \chi_k \chi_l \qquad \frac{N(N-1)(N-2)(N-3)}{12}\ ; \quad \underline{J_{ijkl}} \quad N^4$$

$$2^{N/2} \times 2^{N/2}$$

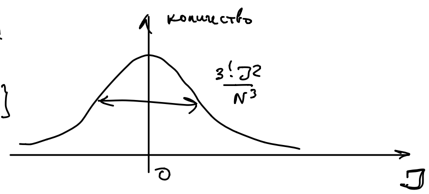
$$\chi_1 = \overset{1}{\sigma_x} \otimes \overset{2}{1\!\!1} \otimes \overset{3}{1\!\!1} \dots \otimes \overset{N}{1\!\!1} \ \} \qquad N = \text{четно}$$

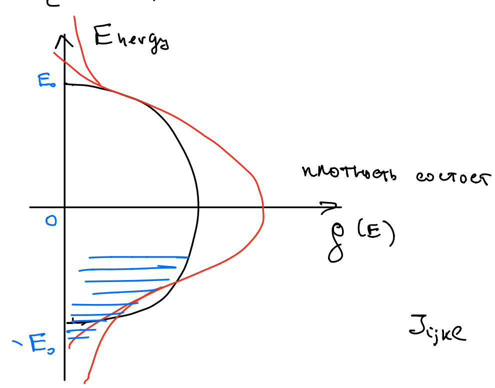
$$\chi_2 = \sigma_y \otimes 1\!\!1 \otimes 1\!\!1 \dots \otimes 1\!\!1 \ \}$$

$$\chi_3 = \sigma_z \otimes \sigma_x \otimes 1\!\!1 \dots 1\!\!1$$

$$\sigma_z \otimes \sigma_z \dots \otimes \sigma_z \otimes \sigma_x \otimes \dots 1\!\!1$$

$$\{ J_{1234},\quad J_{1256}, \dots \quad J_{23\,7\,10} \}$$



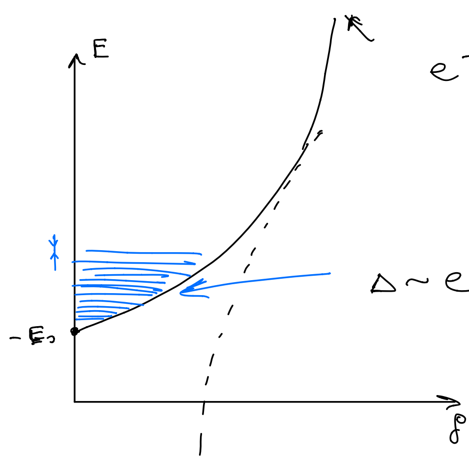


$$H = \sum J_{ijkl}\, c_i^+ c_j^+ c_k c_l \qquad \underline{\underline{'70}}$$

$$\{c_i^+, c_j\} = \delta_{ij}$$

$$\{c_i, c_j\} = \{c_i^+, c_j^+\} = 0$$

$$J_{ijkl} = \int d^3r\, d^3r'\ \psi_i^*(r)\,\psi_j^*(r')\,V(r-r')\,\psi_k(r)\,\psi_l(r')$$



$$e^{-(\log 2)\cdot N} \qquad 2^N \sim e^{(\log 2)\cdot N}$$

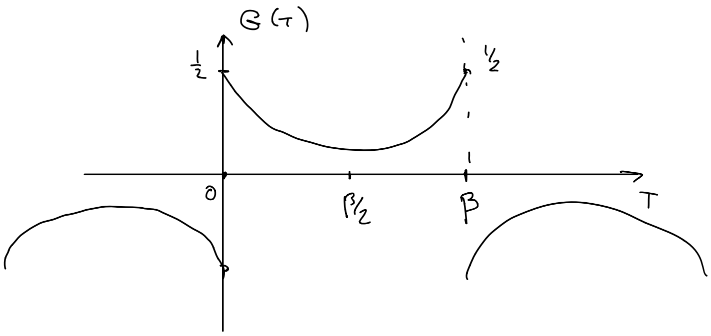
$$\Delta \sim e^{-S_0 \cdot N}; \qquad S_0 = 0.23$$

$$\boxed{N \to \infty}$$

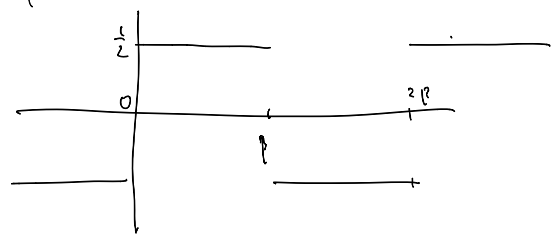
$$G(\tau) = \frac{1}{N}\sum_{i=1}^{N}\langle T\, \chi_i(\tau)\chi_i(0)\rangle_\beta = \begin{cases} \frac{1}{Z}\operatorname{Tr}\left(e^{\tau H}\chi\, e^{-\tau H}\chi\, e^{-\beta H}\right); & \tau > 0 \\ & \tau < 0 \end{cases}$$

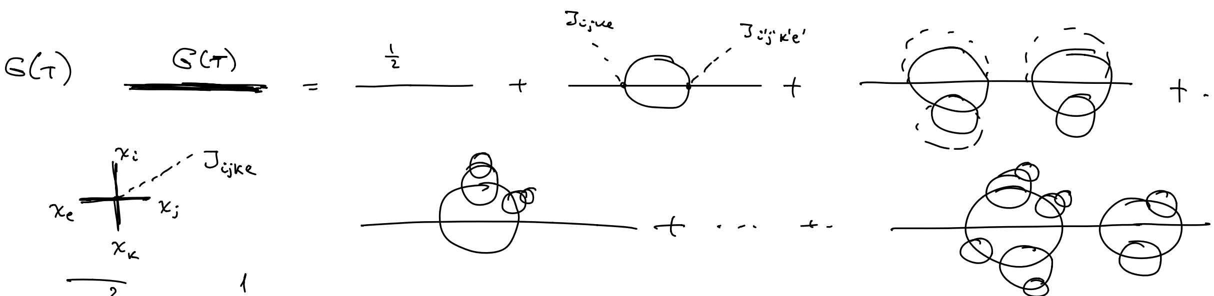
$\tau = 0 \qquad \chi^2 = 1/2 \qquad \{\chi_i, \chi_j\} = \delta_{ij} \qquad G(0^+) = 1/2 \qquad \beta/2$

$G(\beta^-) = 1/2 \qquad \beta = 1/T$

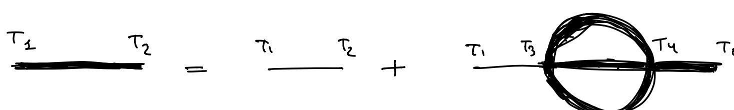


$H = 0 \qquad G_0(\tau) = \frac{1}{2}\operatorname{sgn}\tau$





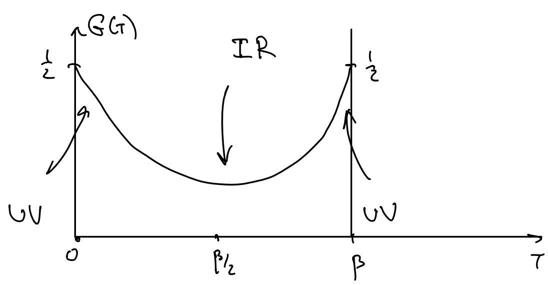
$$\overline{J_{ijkl}^2} = \frac{1}{N^3}$$



$\tau_{12} \equiv \tau_1 - \tau_2$

$$\cancel{G(\tau_{12})} = G_0(\tau_{12}) + J^2\int_0^\beta d\tau_3\, d\tau_4\; G_0(\tau_{13})\, G^3(\tau_{34})\, G(\tau_{42})$$

$\frac{1}{2}\operatorname{sgn}\tau_{13}$

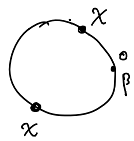


$T \to 0 \qquad \beta \to \infty$

$$\begin{cases} \Sigma(\tau) = J^2 G^3(\tau) \\ G(\omega_n) = \dfrac{1}{\cancel{i\omega_n} - \Sigma(\omega_n)} \end{cases}$$

$$G(\omega_n) = \int_0^\beta d\tau\; G(\tau)\, e^{i\omega_n\tau}$$

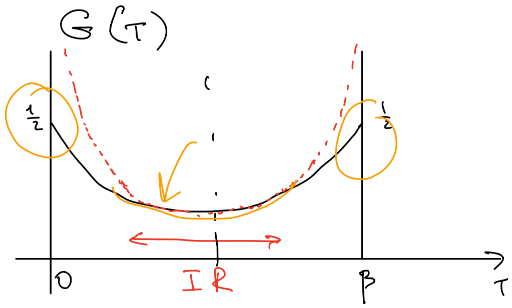
$$\omega_n = \frac{2\pi}{\beta}\left(n + \tfrac{1}{2}\right)$$



$$G(\tau) = \frac{1}{\left(\sin\frac{\pi\tau}{\beta}\right)^{2\Delta}} \qquad \Delta = 1/4$$

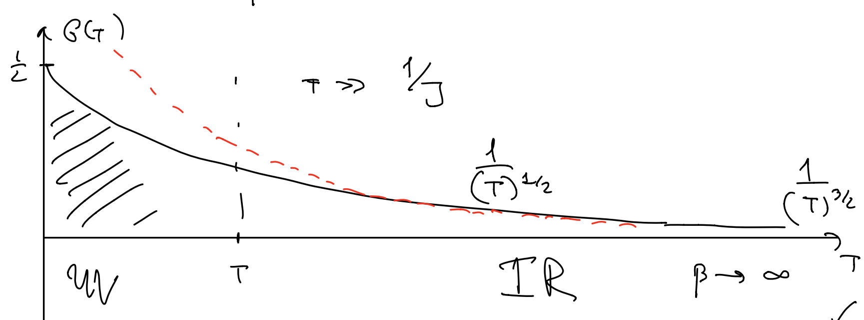
$$(\beta J) \to \infty$$

$$\to \frac{1}{|\tau|^{2\Delta}} = \frac{1}{\tau^{1/2}}$$



$\beta J \gg 1 \qquad \left(N \to \infty \gg \beta J \right)$

$T \to 0 \qquad \beta \to \infty$



$\underline{J = 1}$

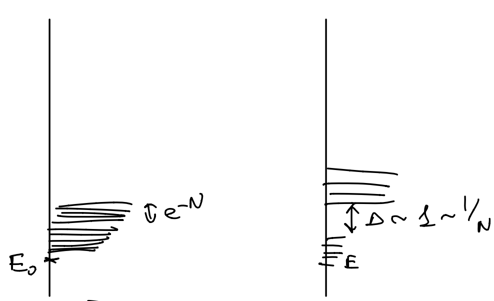
$$G(\tau) = \frac{1}{\left(\sin\frac{\pi\tau}{\beta}\right)^{\frac{1}{2}}} \left(1 + \frac{1}{\underbrace{(\beta J)}} f(\tau) + \cdots \frac{1}{(\beta J)^{n}} f_n(\tau) \right)$$

$$\sum_{n,m} \langle n| e^{\tau H} \chi\, e^{-\tau H} |m\rangle\langle m| \chi\, e^{-\beta H} |n\rangle = \sum_{m,n} |\langle n|\chi|m\rangle|^2 e^{-\beta E_n + \tau(E_n - E_m)}$$

$$\to e^{-\Delta \tau}$$

$$\downarrow$$

$$\Delta \sim e^{-N}$$



$E_0 \sim J$

$$F = -\log Z$$

$$\langle F \rangle_J = \langle \log Z \rangle_J = \lim_{n \to 0} \left\langle \frac{Z^n - 1}{n} \right\rangle_J \qquad n = \mathbb{Z}$$

β

$$Z = \int D\chi_i \, e^{\int_0 d\tau\, \chi_i (\partial_\tau)\chi_i - \int J_{ijkl}\, \chi_i(\tau)\chi_j(\tau)\chi_k(\tau)\chi_l(\tau)}$$

$$Z^n = \int D\chi_i^a \, e^{\int_0^\beta d\tau\, \chi_i^a \partial_\tau \chi_i^a - \int J_{ijkl}\, \chi_i^a \chi_j^a \chi_k^a \chi_l^a} \qquad a = 1, \dots, n$$

$$\int dJ_{ijkl}\, e^{-\frac{J_{ijkl}^2}{2\left(\frac{3!J^2}{N^3}\right)}}\, Z^n =$$

$$= \int D\chi_i^a \, e^{\int_0^T d\tau\, \chi_i^a \partial_\tau \chi_i^a - \frac{J^2}{N^3}\int d\tau d\tau'\, \chi_i^a(\tau)\dots\chi_l^a(\tau)\chi_i^b(\tau')\dots\chi_l^b(\tau')}$$

$$= \int D\chi_i^a \, e^{\int_0^T d\tau\, \chi_i^a \partial_\tau \chi_i^a - \frac{J^2}{N^3}\int d\tau d\tau' \left(\chi_i^a(\tau)\chi_i^b(\tau')\right)^4}$$

$$\int DG_{ab}\, \delta\left(G_{ab}(\tau,\tau') - \chi_i^a(\tau)\chi_i^b(\tau')\right) = \mathbb{1}$$

$$= \int DG_{ab} D\chi_i^a \, e^{\int d\tau\, \chi_i^a \partial_\tau \chi_i^a - \frac{J^2}{N^3}\int d\tau d\tau' \left(G_{ab}(\tau,\tau')\right)^4}\, \delta\left(G_{ab} - \chi_i^a \chi_i^b\right)$$

$$\downarrow$$

$$\int D\Sigma_{ab}\, e^{N(G_{ab} - \chi_i^a\chi_i^b)\Sigma_{ab}}$$

$$=$$

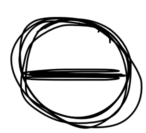
$$\int DG\, D\Sigma\, e^{N\left[\underbrace{\log \mathrm{Pf}\left(\cancel{\partial_\tau} - \Sigma\right) + \int G(\tau,\tau')\Sigma(\tau,\tau') + \frac{J^2}{4}G^4}_{S[G,\Sigma]}\right]}$$

G, Σ - action $\qquad S[G,\Sigma]$

$$\frac{\delta S}{\delta G(\tau,\tau')} = 0 \qquad \frac{\delta S}{\delta \Sigma} = 0$$

$$\downarrow$$

$$\begin{cases} \Sigma = J^2 G^3(\tau) \\ G(\omega) = \dfrac{1}{i\omega - \Sigma(\omega)} \end{cases}$$



G_*, Σ_*

$\uparrow$

$f(\tau)$ репараметрез,

$$G(\tau,\tau') \to \left(f(\tau) f(\tau')\right)^\Delta G(f(\tau), f(\tau')) \qquad \Delta = 1/4$$

$$\Sigma(\tau,\tau') \to \left(f(\tau) f(\tau')\right)^{1-\Delta} \Sigma(f(\tau), f(\tau'))$$

$$S[G,\Sigma] \longrightarrow S[G,\Sigma]$$

$$\partial_\tau - \Sigma \longrightarrow \sigma(\tau,\tau') - \Sigma(\tau,\tau') = \tilde{\Sigma}(\tau,\tau')$$

$$\underline{\sigma(\tau,\tau') = \delta'(\tau-\tau')}$$

$$S[G,\tilde{\Sigma}] = \underbrace{\log \mathrm{Pf}(\tilde{\Sigma}) - \int\left(G\tilde{\Sigma} + \frac{J^2}{4}G^4\right)}_{S_{conf}(G,\tilde{\Sigma})} + \underbrace{\int \sigma(\tau,\tau')\, G(\tau,\tau')\, d\tau\, d\tau'}_{\text{возмущение}}$$

$$G_* \sim \frac{1}{|\tau-\tau'|^{1/2}} \to G_* = \frac{(f'(\tau)\, f'(\tau'))^{1/4}}{|f(\tau)-f(\tau')|^{1/2}}$$

$$\underline{\gamma \int_0^\beta d\tau \left\{ \tan\frac{f(\tau)}{2}, \tau \right\}}$$

$$\{\varphi(\tau),\tau\} = \frac{\varphi'''}{\varphi'} - \frac{3}{2}\left(\frac{\varphi''}{\varphi'}\right)^2$$

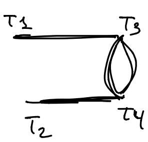
$$S[G,\Sigma] = \log \mathrm{Pf}(\partial_\tau - \Sigma) - \int_0^\beta \left(G\Sigma + \frac{J^2}{4}G^4\right) \qquad \frac{\delta S}{\delta G} = \frac{\delta S}{\delta \Sigma} = 0$$

$$G_*, \Sigma_*$$

$$G = G_* + g; \quad \Sigma = \Sigma_* + \sigma$$

$$S[G,\Sigma] = S[G_*,\Sigma_*] + \int \sigma(\tau_1,\tau_2)\, \Pi(\tau_1,\tau_2,\tau_3,\tau_4)\, \sigma(\tau_3,\tau_4) + \int \sigma g + \int g(\cdot\cdot) g$$

$$= S[G_*,\Sigma_*] + \int g(\tau_1,\tau_2)\left(K(\tau_1,\tau_2,\tau_3,\tau_4) - 1\right) g(\tau_3,\tau_4)\, d\tau_1 \dots d\tau_4$$

$K(\tau_1,\tau_2;\tau_3,\tau_4) =$  $= 3J^2 G_*(\tau_{13})\, G_*(\tau_{24})\, G_*^2(\tau_{34})$

$$\int K(\tau_1,\tau_2;\tau_3,\tau_4)\, \Phi_h(\tau_3,\tau_4)\, d\tau_3\, d\tau_4 = k_h\, \Phi_h(\tau_1,\tau_2)$$

$$K_c = 3J^2 G_c(\tau_{13})\, G_c(\tau_{24})\, G^2(\tau_{34}) \qquad \underline{G_c}(\tau) = \frac{1}{\left(\sin\frac{\pi\tau}{\beta}\right)^{1/2}}$$

$$Z = \int \frac{D\varphi}{\mathrm{Diff}(S^1)}\, e^{\gamma\int_0^\beta d\tau \{e^{i\varphi},\tau\}} \qquad \left(\frac{\varphi'''}{\varphi'} + \left(\frac{\varphi''}{\varphi'}\right)\right)$$

Witten, Stanford

$$\varphi(2\pi) = \varphi(0) + 2\pi$$

$$\langle G(\tau_1,\tau_2)\rangle = \int Df \quad \frac{\left(f'(\tau_1)\, f'(\tau_2)\right)^{\Delta}}{|f(\tau_1)-f(\tau_2)|^{2\Delta}}\; e^{\gamma \int d\tau \{\tan \frac{f(\tau)}{2}, \tau\}} = \cdots$$

$$K_c * \Phi_{h=2} = 1 \cdot \Phi_{h=2} \qquad \int g\,(\underset{0}{K}-1)\, g^{\Phi_{h=2}} = \int$$

$$1 - \frac{c}{\beta J}$$