COVARIANT FERMIONIC VERTEX IN SUPERSTRINGS

V.G. KNIZHNIK
Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Kosygina 2, Moscow, USSR

Received 21 June 1985

A covariant fermionic vertex is constructed in terms of Ramond spinors and ghost operators, starting from the Neveu–Schwarz model. Correlation functions of spinors and ghosts are defined. The supersymmetry algebra is realized as the algebra of massless vertices with zero momentum.

Recently, superstrings have become the focus of a great deal of interest. A serious disadvantage of the existing formulation [1] is the lack of covariance. In ref. [1] only the light-cone analysis has been performed, which obscures the geometrical meaning of superstrings. In this letter I will try to overcome this drawback and present a covariant formulation. The starting point will be the Neveu–Schwarz (NS) model [2]. For simplicity only the open string sector will be considered.

Amplitudes in the NS model are given by integrals of the correlation functions of the vertices $V_i^{\mathrm{f}}(p_i, z_i)$ over their two-dimensional coordinates z_i:

$$A(p_1 \cdots p_N) = \int_{\substack{\text{real,}\\ \text{ordered } z_i}} \prod_i \frac{\mathrm{d}z_i}{d\Omega} \left\langle \prod_{i=1}^{N} V_i^{\mathrm{f}}(p_i, z_i) \right\rangle,$$

$$\mathrm{d}\Omega = (\mathrm{d}z_a \mathrm{d}z_b \mathrm{d}z_c)/z_{ab} z_{bc} z_{ca}, \quad z_{ab} = z_a - z_b, \dots, \tag{1}$$

where the vertices $V_i^{\mathrm{f}}(p_i, z_i)$ are f-components of the superfields

$$V(z, \theta) = V^{\mathrm{b}}(z) + \theta V^{\mathrm{f}}(z).$$

θ is a Grassmann superfield coordinate: $\theta^2 = 0$. To construct a vertex $V(p, z, \theta)$ one can take linear combinations of the superfields

$$\mathrm{D}^{(n_1)} \hat{X}_{\mu_1} \cdots \mathrm{D}^{(n_k)} \hat{X}_{\mu_k} \exp[\mathrm{i} p \hat{X}(z, \theta)]$$

of the equal level numbers $N = \Sigma_{i=1}^{k} n_i$. $\hat{X}_\mu(z, \theta)$ is a superfield of a space–time coordinate of a string world-sheet, normalized as

$$\langle \hat{X}_\mu(z_1, \theta_1) \hat{X}_\nu(z_2, \theta_2) \rangle = -\eta_{\mu\nu} \ln \hat{z}_{12},$$
$$\hat{X}_\mu(z, \theta) = X_\mu(z) + \mathrm{i}\theta \psi_\mu(z),$$
$$\hat{z}_{12} = z_1 - z_2 - \theta_1 \theta_2. \tag{2}$$

D is a covariant derivative: $\mathrm{D} = \partial/\partial\theta + \theta\partial/\partial z$, $\mathrm{D}^2 = \mathrm{d}/\mathrm{d}z$. These linear combinations must satisfy the supergauge conditions

$$G_r V^{\mathrm{b}}(p, z) = 0, \quad n > 0,$$
$$L_0 V^{\mathrm{b}}(p, z) = \tfrac{1}{2} V^{\mathrm{b}}(p, z) = \tfrac{1}{2}(p^2 + N) V^{\mathrm{b}}(p, z), \tag{3}$$

so that (1) be projective invariant and satisfy factorizability. The operators G_r and L_0 in (3) are Laurent components

$$G_r = \oint \frac{\mathrm{d}\xi}{2\pi\mathrm{i}} G(\xi)(\xi - z)^{r+1/2},$$

$$L_n = \oint \frac{\mathrm{d}\xi}{2\pi\mathrm{i}} T(\xi)(\xi - z)^{n+1}, \tag{4}$$

of the supergauge $G(z) = -\mathrm{i}\psi^\mu \partial_z X_\mu(z)$ and conformal generators $T(z) = -\frac{1}{2}\partial_z X^\mu \partial_z X_\mu(z) - \frac{1}{2}\psi^\mu \partial_z \psi_\mu(z)$, satisfying the Neveu–Schwarz superalgebra

$$[L_n, L_m] = (n - m) L_{n+m} + \tfrac{1}{8}\mathscr{D} n(n^2 - 1)\delta_{n+m,0},$$
$$[L_n, G_r] = (\tfrac{1}{2}n - r) G_{n+r},$$
$$\{G_r, G_s\} = 2L_{r+s} + \tfrac{1}{2}\mathscr{D}(r^2 - \tfrac{1}{4})\delta_{r+s,0}. \tag{5}$$

Note, that L_0 measures the conformal dimension of an operator. It is zero for X_μ, $\frac{1}{2}$ for ψ_μ, 1 for ∂_z and $\frac{1}{2}p^2$ for $\exp(\mathrm{i}pX)$. Also $V^{\mathrm{f}} = G_{-1/2}V^{\mathrm{b}}$. In (5) $\mathscr{D}$ is the space–time dimension. The vertices described above correspond to space–time tensors, i.e. bosons. To remove a tachion $p^2 = 1$ one can restrict the model to pure two-dimensional bosonic V^{f}'s, considering only odd levels of N [2].

The model also contains another set of states, discovered by Ramond [3]. The respective operators $\Theta(z)$ are characterized by the property that $\psi^\mu(\zeta)$ changes its sign, when moved around $\Theta(z)$:

$$\psi^\mu(z + (\zeta - z)\mathrm{e}^{2\pi\mathrm{i}})\Theta(z) = -\psi^\mu(\zeta)\Theta(z).$$

Thus $\psi^\mu(\zeta)(\zeta - z)^{1/2}\Theta(z)$ is a single-valued function of ζ in the vicinity of z. We now can introduce operators

$$b_n^\mu\Theta(z) = \oint \frac{\mathrm{d}\zeta}{2\pi\mathrm{i}} \psi^\mu(\zeta)(\zeta - z)^{n-1/2}\Theta(z), \quad n \in \mathbf{Z} \tag{6}$$

and using (2) deduce that they satisfy $\{b_n^\mu, b_m^\nu\} = \eta^{\mu\nu}\delta_{n+m,0}$. The zero mode subalgebra $\{b_0^\mu, b_0^\nu\} = \eta^{\mu\nu}$ enables us to identify b_0^μ with the space–time Dirac matrices $\sqrt{2}\, b_0^\mu = \gamma^\mu$ ensures that $\Theta(z)$ has a spinorial index $A: \Theta \to \Theta^A$, thus being a space–time spinor. A remarkable fact is that if we construct vertices $\Phi(p, z)$ using the Majorana–Weyl Θ^A and such linear combinations of operators

$$\partial_z^{(n_1)}X_{\mu_1} \dots \partial_z^{(n_k)}X_{\mu_k} \mathrm{b}_{-q_1}^{\mu_1} \cdots \mathrm{b}_{-q_m}^{\mu_m}\Theta^A(z) \times \exp[\mathrm{i}pX(z)]$$

of equal levels $M = \Sigma n_i + \Sigma q_i$, that

$$G_r\Phi(p, z) = 0, \quad n \geqslant 0 \tag{7}$$

(note that due to (6) r, s in (4), (5) must be integers), then for $\mathscr{D} = 10$ the number of space–time fermions at each level M will equal the number of bosons at each level $N = 2M + 1$ in the restricted bosonic sector [4]. We will see below that the conformal dimension of Θ^A equals $\mathscr{D}/16$, thus from (3), (5) and (7) one can deduce that the corresponding bosonic and fermionic states have equal masses. On these grounds the space–time supersymmetry of the model has been conjectured in ref. [4].

The difficulty one meets with is that actually the vertices $\Phi(p, z)$, described above, can be used when only two fermions are involved. The expression for the amplitude for $\mathscr{D} = 10$ is [5]:

$$A(p_1, \dots, p_N) = \int \prod_i \frac{\mathrm{d}z_i}{\mathrm{d}\Omega} \left\langle \Phi(p_1, z_1)\Phi(p_2, z_2) \times V^{\mathrm{b}}(p_3, z_3) \prod_{i=4}^{N} V^{\mathrm{f}}(p_i, z_i) \right\rangle \times z_{12}^{-1/4} z_{13}^{-1/2} z_{23}^{-1/2}, \tag{8}$$

where the last factor makes the whole expression projective invariant, i.e. makes all dimensions effectively 1. This shows that to construct a fermionic vertex using Θ's one must understand how to generalize such a factor for the case of more than two fermions. As all formulas must have purely geometrical interpretation, it is natural to interpret such factors as correlation functions of some operators from the sector of Polyakov's ghosts [6].

In order to solve this problem, let's first describe the properties of the correlation functions of chiral Θ's. The adequate point of view is that both ψ_μ and chiral Θ_{L}^A and Θ_{R}^A are highest weights of the vector and spinor representations of the Kac–Moody algebra

$$J_{\mu\nu}(z)J_{\alpha\beta}(z') = -k\frac{\eta_{\mu\alpha}\eta_{\nu\beta} - \eta_{\mu\beta}\eta_{\nu\alpha}}{(z - z')^2} + \frac{\eta_{\nu\alpha}J_{\mu\beta} - \eta_{\nu\beta}J_{\mu\alpha}}{z - z'} + \text{nonsingular terms} \tag{9}$$

of the currents $J_{\mu\nu} = \psi_\mu\psi_\nu$ with central charge $k = 1$. When $\mathscr{D} = 2l$ and $k = 1$ there is a very useful representation of (9) and the currents $J_{\mu\nu}$ in terms of l free bosonic fields φ_a, $a = 1, \dots, l$ [7]. Let us denote $2l(l-1)$ roots of the system D_l by $\boldsymbol{\alpha}$, omitting index, and by $\{\boldsymbol{\alpha}_i\}$ the l vectors of its basis: $\boldsymbol{\alpha}_i^2 = \boldsymbol{\alpha}_{l-1}^2 = \boldsymbol{\alpha}_l^2 = 2$, $\boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_{i+1} = \boldsymbol{\alpha}_{l-2} \cdot \boldsymbol{\alpha}_l = -1$ for $i = 1, \dots, l-2$. Then $\frac{1}{2}\mathscr{D}(\mathscr{D} - 1)$ currents $J_{\mu\nu}$ can be represented by linear combinations of

$$J_{\boldsymbol{\alpha}}(z) = \exp[(\mathrm{i}/\sqrt{2})\boldsymbol{\alpha}\boldsymbol{\varphi}(z)], \quad H_j(z) = (\mathrm{i}/\sqrt{2})\boldsymbol{\alpha}_j\partial_z\boldsymbol{\varphi}(z), \tag{10}$$

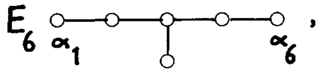
where $\langle\varphi_a(z)\varphi_b(z')\rangle = -\delta_{ab}\ln(z-z')$. The components of $\Theta^A_{\mathrm{L,R}}$ and ψ^μ can also be bosonized in terms of linear combinations of exponents

$$\psi_\mu \leftrightarrow \exp\left[(\mathrm{i}/\sqrt{2})\boldsymbol{\varphi}\boldsymbol{\omega}_{\mathrm{v}}\right],$$
$$\Theta^A_{\mathrm{L,R}} \leftrightarrow \exp\left[(\mathrm{i}/\sqrt{2})\boldsymbol{\varphi}\boldsymbol{\omega}_{\pm}\right] \tag{11}$$

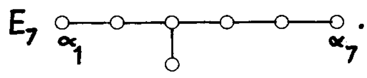
where $\boldsymbol{\omega}_\pm$ and $\boldsymbol{\omega}_{\mathrm{v}}$ are the weights (indices omitted) of the spinor and vector representations of so(2l). The characteristic feature of these systems of weights is that *all* weights in each system have equal lengths. These weights are vectors on a dual lattice with the basis $\{\boldsymbol{\omega}_i, i=1,\dots,l\}$ formed by the fundamental weights: $2\boldsymbol{\omega}_i\boldsymbol{\alpha}_j/\boldsymbol{\alpha}_j^2=\delta_{ij}$. The highest weights $\boldsymbol{\omega}^{\mathrm{h}}$ of each of the above representations are $\boldsymbol{\omega}^{\mathrm{h}}_+=\boldsymbol{\omega}_{l-1}$, $\boldsymbol{\omega}^{\mathrm{h}}_-=\boldsymbol{\omega}_l$ and $\boldsymbol{\omega}^{\mathrm{h}}_{\mathrm{v}}=\boldsymbol{\omega}_1$. The conformal dimension Δ of Θ can be easily calculated from (11) by means of the representation

$$\boldsymbol{\alpha}_1=\boldsymbol{\epsilon}_1-\boldsymbol{\epsilon}_2,\dots,\boldsymbol{\alpha}_{l-1}=\boldsymbol{\epsilon}_{l-1}-\boldsymbol{\epsilon}_l,$$
$$\boldsymbol{\alpha}_l=\boldsymbol{\epsilon}_{l-1}+\boldsymbol{\epsilon}_l,$$
$$\boldsymbol{\omega}_1=\boldsymbol{\epsilon}_1, \boldsymbol{\omega}_{l-1}=\tfrac{1}{2}(\boldsymbol{\epsilon}_1+\cdots+\boldsymbol{\epsilon}_{l-1}-\boldsymbol{\epsilon}_l),$$
$$\boldsymbol{\omega}_l=\tfrac{1}{2}(\boldsymbol{\epsilon}_1+\cdots+\boldsymbol{\epsilon}_l), \tag{12}$$

where $\{\boldsymbol{\epsilon}_i\}$ is an orthonormal basis of $\mathbf{R}^l$: $\boldsymbol{\epsilon}_i\boldsymbol{\epsilon}_j=\delta_{ij}$, and equals $\Delta=\frac{1}{8}l=\frac{1}{16}\mathscr{D}$. An arbitrary weight $\boldsymbol{\omega}_+(\boldsymbol{\omega}_-)$ can be obtained from $\boldsymbol{\omega}_{l-1}(\boldsymbol{\omega}_l)$ when an even number of signs of the $\boldsymbol{\epsilon}$ is changed. Note, that such a construction can be used for any Lie algebra whose simple roots are of equal length and for representations with all weights of equal length. The set of highest weights of representations meeting this condition is: $\boldsymbol{\omega}_1,\dots,\boldsymbol{\omega}_l$ for A_l; $\boldsymbol{\omega}_1, \boldsymbol{\omega}_{l-1}, \boldsymbol{\omega}_l$ for D_l; $\boldsymbol{\omega}_1, \boldsymbol{\omega}_6$ for E_6 and $\boldsymbol{\omega}_7$ for E_7, with the notations:



,



.

The corresponding correlation functions are those of the exponents (11) of the free fields φ_a, thus being trivial. All of them and of course the dimensions coincide with those that can be obtained by means of the methods of ref. [8].

From (11) it follows that for $\mathscr{D}=4k+2$

$$\Theta^A_{\mathrm{L}}(z)\Theta^B_{\mathrm{L}}(z')=(z-z')^{1/2-\mathscr{D}/8}$$
$$\times\left[(\gamma_\mu C)^{AB}\psi^\mu(z')+O^{AB}_{\mathrm{v}}(z-z')\right]$$
$$\Theta^A_{\mathrm{L}}(z)\Theta^B_{\mathrm{R}}(z')=(z-z')^{-\mathscr{D}/8}$$
$$\times\left[C^{AB}+O^{AB}_{\mathrm{s}}(z-z')\right]$$
$$\langle\Theta^A_{\mathrm{L}}(z_1)\Theta^B_{\mathrm{L}}(z_2)\psi_\mu(z_3)\rangle$$
$$=(\gamma_\mu C)^{AB}z_{12}^{1/2-\mathscr{D}/8}z_{13}^{-1/2}z_{23}^{-1/2} \tag{13}$$

where $O^{AB}_{\mathrm{v,s}}(z)$ are analytic single-valued functions of z, zero at $z=0$. C^{AB} is a charge conjugation matrix. For $\mathscr{D}=4k$ one must change the chirality of Θ^A in (13). The "covariantization" of the correlation functions of the Θ^A is a straightforward but not very easy task in general. We will give two examples: for $\mathscr{D}=6$

$$\left\langle\prod_{i=1}^{4}\Theta^{A_i}_{\mathrm{L}}(z_i)\right\rangle$$
$$=\prod_{i<j}(z_{ij})^{-1/4}(\gamma_\mu C)^{A_1A_2}(\gamma^\mu C)^{A_3A_4},$$

and for $\mathscr{D}=10$

$$\left\langle\prod_{i=1}^{4}\Theta^{A_i}_{\mathrm{L}}(z_i)\right\rangle$$
$$=\prod_{i<j}(z_{ij})^{-3/4}\left[z_{14}z_{23}(\gamma_\mu C)^{A_1A_2}(\gamma^\mu C)^{A_3A_4}\right.$$
$$\left.-(\gamma_\mu C)^{A_1A_4}(\gamma^\mu C)^{A_2A_3}\right],$$
$$z_{ij}=z_i-z_j. \tag{14}$$

Keeping these results in mind let us now proceed to the amplitudes, beginning with the amplitudes for massless particles. We will actually seek for a fermionic vertex $\bar{\mathrm{u}}_A(p)F_A(p,z)$ where the Majorana–Weyl c-number spinor amplitude $\mathrm{u}(p)$ satisfies the Dirac equation

$$\hat{p}\mathrm{u}(p)=0, \quad \hat{p}=p^\mu\gamma_\mu. \tag{15}$$

The vector vertex is known to be

$$\zeta^\mu V^{\mathrm{f}}_\mu(p,z)=(\partial/\partial\theta)\{\mathrm{i}\zeta^\mu\mathrm{D}\hat{X}_\mu\exp[\mathrm{i}p\hat{X}(z,\theta)]\}$$
$$=\zeta^\mu(\mathrm{i}\partial_z X_\mu-\psi_\mu p\cdot\psi)\exp[\mathrm{i}pX(z)]$$
$$=G_{-1/2}\{-\zeta\cdot\psi\exp[\mathrm{i}pX(z)]\}. \tag{16}$$

It is also known that two of the vector vertices in a multivector amplitude can be represented in the b-form (the so-called $\mathscr{F}_2$-formalism, see ref. [5]):

$$\zeta^\mu V_\mu^{\mathrm{b}}(p,z) = -\zeta^\mu \psi_\mu \exp[\mathrm{i}pX(z)]\chi(z). \tag{17}$$

A ghost fermion $\chi(z)$ of dimension $\frac{1}{2}$ must be introduced in order to maintain projective invariance:

$$\langle \chi(z)\chi(z')\rangle = (z-z')^{-1}.$$

Now one can see that in (8) each fermionic vertex can be represented by

$$\bar{\mathrm{u}}_A F_A(p,z) = \bar{\mathrm{u}}\Theta_{\mathrm{L}} \exp[\mathrm{i}pX(z)]\,\sigma(z), \tag{18}$$

where $\sigma(z)$ is an additional ghost operator of dimension $\frac{3}{8}$, such that

$$\langle \sigma(z_1)\sigma(z_2)\chi(z_3)\rangle = z_{12}^{-1/4} z_{13}^{-1/2} z_{23}^{-1/2}. \tag{19}$$

The vertex (18) is thus a "square root" from one of the two b-form vertices (17) in the sense that

$$\bar{\mathrm{u}}_1 F_{p_1}(z_1)\bar{\mathrm{u}}_2 F_{p_2}(z_2) = z_{12}^{p_1 p_2 - 1} \times\left[-2\bar{\mathrm{u}}_1\gamma_\mu \mathrm{u}_2 V^\mu_{p_1+p_2}(z_2) + \mathrm{O}(z_{12})\right]. \tag{20}$$

In (20) we have restored the correct normalization. The vector, obtained on the RHS of (20) is transverse due to (15), and we have a pole in the amplitude when it is on-shell: $p_1 p_2 = 0$. Another useful relation

$$\zeta_\mu V^\mu_{p_1}(z_1)\bar{\mathrm{u}}_2 F_{p_2}(z_2) = z_{12}^{p_1 p_2 - 1} \times\left[\tfrac{1}{2}\bar{\mathrm{u}}_2 \hat{\zeta}(\hat{p}_1+\hat{p}_2)F_{p_1+p_2}(z_2) + \mathrm{O}(z_{12})\right] \tag{21}$$

is valid both for the b- and for f-components (16), (17).

Now, it is obvious that the remaining b-form vertex in (8) can be divided in an analogous fashion. The obtained correlation functions contain four fermionic vertices (18) and a number of f-form vector vertices (16), the relevant 4σ-correlation function being

$$\left\langle \prod_{i=1}^{4}\sigma(z_i)\right\rangle = \prod_{i<j} z_{ij}^{-1/4}. \tag{22}$$

Together with (14) this immediately yields the correct result for the four-fermion amplitude [1]. Yet, the simplest way to obtain this and also the two-fermion amplitude without knowledge of (14), (22) is to use the superstring vertex algebra (SVA) (20), (21) directly, evaluating correlation functions by means of the bootstrap procedure. The fundamental fact is that the correlation function becomes single-valued in all arguments z_i when multiplied by $\prod_{i<j}(z_{ij})^{-p_i p_j}$, and for each i falls off like z_i^{-2} when $z_i \to \infty$, After this operation the SVA becomes part of some unknown Kac–Moody-like algebra, and the unique three-fermion Jacobi identity, that can be verified, leads to the famous condition of the existence of super-Yang–Mills theories [4]:

$$\gamma_\mu \mathrm{u}_3\bar{\mathrm{u}}_1\gamma^\mu \mathrm{u}_2 + \gamma_\mu \mathrm{u}_1\bar{\mathrm{u}}_2\gamma^\mu \mathrm{u}_3 + \gamma_\mu \mathrm{u}_2\bar{\mathrm{u}}_3\gamma^\mu \mathrm{u}_1 = 0. \tag{23}$$

To construct amplitudes with more than four fermions one must divide f-form vector vertices (16) in the four-fermion and vectors correlation functions constructed above. For the simplest case of a six-fermion amplitude one can easily guess that five fermions can be taken in the form (18) and the sixth in a different form:

$$\bar{\mathrm{u}}_6 F'_{p_6}(z) = G_1[\bar{\mathrm{u}}_6\Theta \exp(\mathrm{i}pX(z))]\,\Omega(z), \tag{24}$$

where $\Omega(z)$ is a new ghost operator of negative dimension $-\frac{5}{8}$, such that

$$\left\langle \prod_{i=1}^{5}\sigma(z_i)\Omega(z)\right\rangle = \prod_{i<j}(z_{ij})^{-1/4}\prod_{i=1}^{5}(z_i - z)^{1/4}. \tag{25}$$

The constructed six-fermion function will have correct factorization properties in the $p_i + p_6$ channels, as is clear from

$$\bar{\mathrm{u}}_i F_{p_i}(z_i)\bar{\mathrm{u}}_6 F'_{p_6}(z_6) = z_{i6}^{p_i p_6 - 1} \times\left[\bar{\mathrm{u}}_i\gamma^\mu \mathrm{u}_6 V_\mu^{\mathrm{f}}(p_i + p_6, z_6) + \mathrm{O}(z_{i6})\right]. \tag{26}$$

Thus (26) is the sought for division of (16) and the obvious generalization to the case of $N = 2k + 4$ fermions and M vectors is

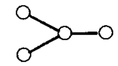
$$\mathscr{F}_{\mathrm{N,M}} = \left\langle \prod_{i=1}^{k+4}\bar{\mathrm{u}}_i F_{p_i}(z_i)\prod_{j=1}^{k}\bar{\mathrm{u}}_j F'_{p_j}(\xi_j) \times \prod_{l=1}^{M}\zeta_l^\mu V_\mu^{\mathrm{f}}(p_l, z_l)\right\rangle, \tag{27}$$

where the relevant ghost correlation function is

$$\left\langle \prod_{i=1}^{k+4}\sigma(z_i)\prod_{j=1}^{k}\Omega(\xi_j)\right\rangle = \prod_{i<j}(z_{ij})^{-1/4}\prod_{l<m}\xi_{lm}^{-1/4}\prod_{i,l}(z_i - \xi_l)^{1/4}. \tag{28}$$

One can prove that (27) is symmetric in all fermions up to total derivatives, which are irrelevant due to the "cancelled propagator argument", but the proof is cumbersome. To perform it, it is necessary that only the gauge conditions (3), (7) should be satisfied, where in the case of (27) $\Phi(p,z) = \bar{u}\Theta \exp[ipX(z)]$ and $V^{b}(p,z) = \zeta^{\mu} V_{\mu}^{b}(P,z)$. Thus these expressions can be replaced by arbitrary states, satisfying (3), (7). Actually, the only nonzero correlations of σ and Ω are (25). Thus there is a unified expression for a fermionic vertex for an arbitrary state satisfying (7):

$$F(p,z) = \Phi(p,z)\sigma(z) + \Omega(z) G_{-1}\Phi(p,z). \tag{29}$$

When one proceeds to the light-cone gauge, ghosts disappear, the dimension of Θ^{A} reduces from $\frac{5}{8}$ to $\frac{1}{2}$ and the Θ^{A} of the same chirality are free fermions with respect to each other, because the representations ψ^{μ}, Θ_{L}^{A} and Θ_{R}^{A} of (9) are equivalent, as follows from the symmetry of the Dynkin diagram for D_8: 

The vertex (29) turns into the one described in ref. [1].

We will now discuss supersymmetry of the amplitudes obtained. For this purpose let us consider the subalgebra of the SVA of vertices with zero momentum. Identifying the operator

$$\oint \frac{d\xi}{2\pi i} V_0^{\mu}(\xi) = \hat{p}^{\mu},$$

with a space–time translation operator (the contour surrounds the vertex whose transformation is under study), one sees that

$$\hat{Q}^{A} = \oint \frac{d\xi}{2\pi i} F_0^{A}(\xi)$$

satisfies the space–time supersymmetry algebra

$$\{\hat{Q}^{A}, \hat{Q}^{B}\} = -2(\gamma^{\mu}C)^{AB}\hat{p}_{\mu}. \tag{30}$$

From the SVA it also follows that fermionic and bosonic vertices are correctly transformed by $\hat{Q}^{A}$ into each other. Let us now consider the function

$$\left\langle \epsilon_A F_0^{A}(z) \prod_i W(p_i, z_i) \right\rangle, \tag{31}$$

where W represents both bosonic and fermionic vertices. This function is analytic in z and has only simple poles at $z = z_i$. The conformal dimension of $F_0^{A}(z)$ is 1. Thus when $z \to \infty$, (31) is $\sim z^{-2}$, i.e. the sum of the residues is zero:

$$\sum_i \left\langle \epsilon_A \hat{Q}^{A} W(p_i, z_i) \prod_{j \neq i} W(p_j, z_j) \right\rangle = 0.$$

But this is nothing else but the supersymmetry condition of the corresponding amplitude. Our conclusion is that the supersymmetry algebra is realized as the algebra of massless vertices.

Formulas (8), (28), (29) together with the SVA [‡1] completely describe all covariant amplitudes in superstring theory and its supersymmetry.

The proofs of the symmetry of (27), and of the fact that $\sigma(z)$ and $\Omega(z)$ are really representations of superconformal field theory of the fermionic string ghosts, will be published elsewhere. An interesting question is how to obtain (29) on purely geometrical grounds.

I'm grateful to A. Polyakov for a lot of useful conversations. Actually a study of SVA has been initiated by his suggestion.

Note Added. After this work had been completed ref. [9] containing similar results was obtained.

‡1 I must mention, that one cannot use the SVA when both fermions in (20) or a fermion in (21) are in the form (24). But any given set of 1, 2 or 3 fermions can always be represented by (18) and for them the SVA will be valid. That is all we need for deriving (23) and (30).

References

[1] J.H. Schwarz, Phys. Rep. 89C (1982) 223.
[2] A. Neveu and J.H. Schwarz, Nucl. Phys. B31 (1971) 86.
[3] P. Ramond, Phys. Rev. D3 (1971) 2415.
[4] F. Gliozzi, J. Scherk and D.I. Olive, Nucl. Phys. B122 (1977) 253.
[5] J. Scherk, Rev. Mod. Phys. 47 (1975) 123, and references therein.
[6] A.M. Polyakov, Phys. Lett. 103B (1981) 211.
[7] I.B. Frenkel and V.G. Kac, Inv. Math. 62 (1980) 23; G. Segal, Commun. Math. Phys. 80 (1981) 301.
[8] V.G. Knizhnik and A.B. Zamolodchikov, Nucl. Phys. B247 (1984) 83.
[9] D. Friedan, S. Shenker and E. Martinec, EFI preprint-85-32.