EXACT SOLUTION OF THE TWO-DIMENSIONAL MODEL WITH ASYMPTOTIC FREEDOM

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An exact solution of the SU(2)-symmetric theory with four-fermion interaction in one spatial and one time dimension is derived

In this article we study the SU(2)-symmetric theory with four-fermion interaction in one spatial and one time dimension and give an exact solution of it We restrict ourselves to the massless case The interaction lagrangian is of the form

$$L_\mathrm{I} = -\tfrac{1}{2}g_\mathrm{s}(\bar{\Psi}\gamma_\mu\Psi)^2 - \tfrac{1}{2}g_\mathrm{v}(\bar{\Psi}\gamma_\mu\tau^a\Psi)^2 \tag{1}$$

Ψ is the doublet of Fermi fields, τ^a are the isotropic matrices, γ_μ the Dirac matrices $\gamma_0 = \sigma^x$, $\gamma_1 = i\sigma^y$, $\gamma_5 = \sigma^z$ The theory is interesting since, due to the absence of bare mass in the chosen interaction, it possesses continuous γ_5 invariance The presence of the infrared catastrophe [1] for $g_\mathrm{v} > 0$ evidently leads to the spontaneous appearance of mass [2,3]

The model is also of interest since it is dual to the theory of the three-component $\boldsymbol{n}$-field with the action

$$S = (1/2g_0^2)\int(\partial_\mu \boldsymbol{n})^2\,\mathrm{d}^2x\,,$$

like the massive Thirring model is dual to the two-component $\boldsymbol{n}$-field As has been shown by Berezinskii this system, with the vortices taken into account, is equivalent to the Coulomb gas, which is in the plasma phase if the temperature is above the critical value

Coleman has proven the equivalence of the partition function of the Coulomb gas and the massive Thirring model (the exact equivalence should be sought in the vicinity of the critical value of the temperature when vortices with circulations >1 are not essential)

The three-component $\boldsymbol{n}$-field model (SU(2) symmetry) and the more general case of the CP($N-1$) model (SU(N) symmetry) are, as has been pointed out, two-dimensional analogues of the gauge theories Wonderful properties of the chiral and fermion theories in 1 + 1 dimensions are the hidden symmetry and the complete integrability discovered on the classical and quantum levels So far the hidden symmetry has not been discovered in gauge theories However, if we believe in the two-dimensional analogy, its existence is beyond doubt The exact solution given in this article is based on the hidden symmetry which the theory with the interaction of the form (1) possesses in the massless case and that is why it is interesting The method of solving the model is closely related to the ideas of Onsager and Baxter [4], Yang [5], Gaudin [6], Berezin and Sushko [7,8], Faddeev et al [10,11] and Kulish [12,1:

In this article we will find wave eigenfunctions and energy levels of the hamiltonian in terms of pseudoparticles in a finite volume In order to find the physical spectrum it is necessary to let the volume tend to infinity, to introduce a boundary momentum, to fill the Dirac sea and to perform dimensional transmutation

The total hamiltonian corresponding to (1) is, in second quantization representation, of the form

$$\hat{H} = \int_x \{-i\Psi^{+\alpha}_{i_1}(x)\sigma^z_{i_1 i_2}(\partial/\partial x)\Psi^\alpha_{i_2}(x)$$

$$+\tfrac{1}{2}\Psi^{+\alpha_1}_{i_1}(x)\Psi^{+\beta_1}_{j_1}(x)\,[g_\mathrm{s}\delta_{\alpha_1\alpha_2}\delta_{\beta_1\beta_2} + g_\mathrm{v}\tau^a_{\alpha_1\alpha_2}\tau^a_{\beta_1\beta_2})$$

$$\times(\delta_{i_1 i_2}\delta_{j_1 j_2} - \sigma^z_{i_1 i_2}\sigma^z_{j_1 j_2})]\,\Psi^{\alpha_2}_{i_2}(x)\Psi^{\beta_2}_{j_2}(x)\}$$

$$\times\ \{\Psi^\alpha_i(x)\Psi^{+\beta}_j(y)\} = \delta_{\alpha\beta}\delta_{ij}\delta(x-y) \tag{2}$$

Here the Latin indices i, j denote the Dirac indices, the

Greek indices α, β denote isotopic components of spinor fields The complete symmetry group of (2) is the group $U(1) \times SU(2) \times U(1)_5$ of global transformations By virtue of this fact the overall number of fermions (pseudoparticles) is conserved

$$\hat{N} = \int \Psi^+ \Psi \, dx \,, \tag{3}$$

and the difference of the number of right-handed and left-handed fermions is

$$\hat{N}_5 = \int \Psi^+ \sigma^z \Psi \, dx \tag{4}$$

Evidently hamiltonian (2) has a pseudovacuum eigenvector, $|0\rangle$, satisfying the equations

$$\Psi_i^{\alpha}(x)|0\rangle = 0 \tag{5}$$

By virtue of the conservation of the number of pseudoparticles $\hat{N}$ the eigenfunctions of (2) may be sought with a fixed value of $\hat{N}$

$$|N\rangle = \int dx_1 \quad dx_N \tag{6}$$

$$\times f^{i_1 \quad i_N}_{\alpha_1 \quad \alpha_N}(x_1, \quad , x_N) \Psi^{+\alpha_1}_{i_1}(x_1) \quad \Psi^{+\alpha_N}_{i_N}(x_N)|0\rangle$$

The equation $\hat{H}|N\rangle = E|N\rangle$ is equivalent to the following equation for the function $f^{i_1 \quad i_N}_{\alpha_1 \quad \alpha_N}(x_1, \quad , x_N)$ which is an equation for N fermions, interacting with a δ-function pair potential

$$\left\{ \sum_{n=1}^{N} -i\sigma_n^z \frac{\partial}{\partial x_n} + \tfrac{1}{2} \sum_{n<m} (g_s + g_v \boldsymbol{\tau}_n \boldsymbol{\tau}_m)(1 - \sigma_n^z \sigma_m^z) \right.$$

$$\left. \times \, \delta(x_n - x_m) \right\} f = Ef \tag{7}$$

σ_n^z and τ_n^a, as regular spin matrices, act on the Dirac indices i_n and the isotopic indices α_n of the wavefunction, respectively They act on the remaining indices as unit operators

Apart from eq (7) the wavefunction obeys the antisymmetry requirements

$$f^{\alpha_{q_1} \quad \alpha_{q_N}}_{i_{q_1} \quad i_{q_N}}(x_{q_1}, \quad , x_{q_N})$$

$$= (-1)^{\eta_Q} f^{\alpha_1 \quad \alpha_N}_{i_1 \quad i_N}(x_1, \quad , x_N) \tag{8}$$

Here $Q \equiv (q_1, \quad , q_N)$ is a certain permutation of $(1, \quad , N)$, η_Q is the parity of the permutation

The last requirements for the wavefunction are the periodic boundary conditions

$$f(\quad , x_n + L, \quad) = f(\quad , x_n, \quad) \tag{9}$$

Let us divide the space of the coordinates x_n into a number of regions which will be denoted by $Q = (q_1, \quad , q_N)$

$$Q \quad x_{q_1} < x_{q_2} < \quad < x_{q_N} \tag{10}$$

The value of the wavefunction in the region Q is denoted by $f^Q(x_1, \quad , x_N)$ Since in each region the coordinates do not coincide, the wavefunction obeys by virtue of eq (7) the free N-particle Dirac equation If for the time being we ignore the antisymmetry condition (8) the wavefunction in a certain region, e g $Q = I$, i e $I = (1, \quad , N)$ and $x_1 < \quad < x_N$, may be chosen as a superposition of plane waves

$$f^I(x) = \sum_P (-1)^{\eta_P} A^P_{\alpha_1 \quad \alpha_N}$$

$$\times u_{i_1}(\sigma_{p_1}) \quad u_{i_N}(\sigma_{p_N}) \exp(i x_n K_{p_n}) \tag{11}$$

Here

$$u_i(1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad u_i(-1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$K_1, \quad , K_N$ is a fixed set of momenta, $P = (p_1, \quad , p_N)$ is a permutation of $(1, \quad , N)$, $A^P_{\alpha_1 \quad \alpha_N}$ are so far arbitrary coefficients and $\sigma_1, \quad , \sigma_N$ are helicities taking on the values ± 1

If we restrict ourselves to the two-particle sector $N = 2$, then due to the conservation of energy and momentum the wavefunction in the region $Q = (2, 1)$, i e , $x_2 > x_1$, will also be a superposition of plane waves with the same momenta K_1 and K_2 The Bethe hypothesis, which will be proven below, states that even if $N > 2$ a set of momenta in all regions Q will be retained It is clear that matching of the wavefunctions for coinciding coordinates ($x_n = x_m$) imposes rigid conditions of self-consistency These conditions are equivalent to the factorization equations of the two-particle S matrix of the pseudoparticles It should be mentioned that this condition is observed only if the bare mass $m_0 = 0$ This occurs for any value of the coupling constants g_s and g_v

When the antisymmetry conditions are taken into account, the wavefunction f^Q in the region $x_{q_1} < \quad < x_{q_N}$ can be given in terms of the coefficients A^P_{α} of the wavefunction f^I by the following expression

$$f^Q = \sum_P (-1)^{\eta_P} A^{QP}_{\alpha_{q_1} \dots \alpha_{q_N}}$$
$$\times u_{i_1}(\sigma_{p_1}) \dots u_{i_N}(\sigma_{p_N}) \exp(i x_n K_{p_n}) \quad (12)$$

QP denotes the product of permutations

Let us now find the conditions which arise from eq (7) when a pair of coordinates coincide For the two-particle case, we arrive at the relation

$$A^{12}_{\alpha_1\alpha_2} = b(\tfrac{1}{2}(\sigma_1 - \sigma_2))A^{21}_{\alpha_1\alpha_2}$$
$$+ a(\tfrac{1}{2}(\sigma_1 - \sigma_2))A^{21}_{\alpha_2\alpha_1} \equiv \hat{K}_{12} A^{21}_{\alpha_1\alpha_2}, \quad (13)$$

$$\hat{K}_{12} = b(\sigma_{12}) + a(\sigma_{12})P_{12},$$

$$P_{12} = \tfrac{1}{2}(1 + \boldsymbol{\tau}_1 \boldsymbol{\tau}_2), \quad \sigma_{12} \equiv (\sigma_1 - \sigma_2)$$

The symbols used here are as follows σ_1 and σ_2 are the helicities of the pseudoparticles taking on the values ± 1, P_{12} is the operator of interchange of the isotopic indices,

$$b(\sigma) = \frac{1}{2}\left[\frac{\sigma + i\lambda_1}{\sigma - i\lambda_1} - \frac{1 + i\sigma\lambda_0}{1 - i\sigma\lambda_0}\right], \quad (14)$$

$$a(\sigma)/b(\sigma) = -i\,[(1 + \lambda_1\lambda_0)/(\lambda_1 - \lambda_0)]\,\sigma, \quad (15)$$

$(1/i)\ln[(\sigma + i\lambda_1)/(\sigma - i\lambda_1)]$ and $(1/i)\ln[(1 + i\sigma\lambda_0)/(1 - i\sigma\lambda_0)]$ are the scattering phases for the two particles with total isotopic spin 1 and 0, respectively The quantities λ_1 and λ_0 are given in terms of the coupling constants as

$$\lambda_1 = \mathrm{tg}\,\tfrac{1}{2}(g_v + g_s), \quad \lambda_0 = \mathrm{tg}\,\tfrac{1}{2}(g_s - 3g_v) \quad (16)$$

Let us now dwell upon the case $N = 3$ In this case at $x_1 = x_2 < x_3$ etc a lot of matching conditions appear If we take into account the unitarity condition of the $\hat{K}$ matrix

$$\hat{K}_{12}\hat{K}_{21} = 1, \quad (17)$$

then there are six independent equations

$$A^{123} = K_{23}(\sigma_{23})A^{132}, \quad A^{231} = K_{23}(\sigma_{31})A^{213},$$
$$A^{312} = K_{23}(\sigma_{12})A^{321}, \quad A^{231} = K_{12}(\sigma_{23})A^{321},$$
$$A^{312} = K_{12}(\sigma_{31})A^{132}, \quad A^{123} = K_{12}(\sigma_{12})A^{213} \quad (18)$$

By means of these relations we can express all the coefficients $A^{p_1p_2p_3}_{\alpha_1\alpha_2\alpha_3}$ in terms of $A^{123}_{\alpha_1\alpha_2\alpha_3}$ We can do this in several ways, e g , we can express the coefficient A^{321} by two sequences $(123) \to (132) \to (312) \to (321)$ and $(123) \to (213) \to (231) \to (321)$ It is clear that, in order to fulfill the Bethe hypothesis, the result in the both cases should be the same and the following relations should hold

$$K_{23}(\sigma_{23})K_{12}(\sigma_{13})K_{23}(\sigma_{12})$$
$$= K_{12}(\sigma_{12})K_{23}(\sigma_{13})K_{12}(\sigma_{23}) \quad (19)$$

Below we will need the operator $S_{12}(\sigma)$, expressed in terms of $\hat{K}_{12}$

$$\hat{S}_{12}(\sigma_{12}) = \hat{K}_{12}(\sigma_{12})P_{12} = a(\sigma_{12}) + b(\sigma_{12})P_{12} \quad (20)$$

For $\hat{S}_{12}$ the relations (19) take the form

$$\hat{S}_{12}(\sigma_{12})\hat{S}_{13}(\sigma_{13})\hat{S}_{23}(\sigma_{23})$$
$$= \hat{S}_{23}(\sigma_{23})\hat{S}_{13}(\sigma_{13})\hat{S}_{12}(\sigma_{12}) \quad (21)$$

Substitution of the explicit form of $\hat{S}_{ik}(\sigma_{ik})$ into eq (21) leads to the equation

$$a(\sigma_{12})/b(\sigma_{12}) + a(\sigma_{23})/b(\sigma_{23}) = a(\sigma_{13})/b(\sigma_{13}) \quad (22)$$

It is evident from the form of the functions $a(\sigma)$ and $b(\sigma)$ in eqs (14), (15) that eq (22) is fulfilled identically Note that if $m_0 \neq 0$, a and b would be hyperbolic functions of the rapidities of the pseudoparticles and eq (22) could not be fulfilled since it requires a linear dependence of the ratio a/b on the argument

$$a(\sigma)/b(\sigma) = \mathrm{const} \cdot \sigma \quad (23)$$

In the general case of an arbitrary number of particles the relations for the coefficients $A^{p_1 \dots p_N}_{\alpha_1 \dots \alpha_N}$ are of the form

$$A^{\dots p_n p_{n+1} \dots}_{\dots \alpha_n \alpha_{n+1} \dots} = \hat{S}_{n\,n+1}(\tfrac{1}{2}(\sigma p_n - \sigma p_{n+1}))A^{\dots p_{n+1} p_n \dots}_{\dots \alpha_{n+1} \alpha_n \dots} \quad (24)$$

Now all the coefficients $A^{p_1 \dots p_N}_{\alpha_1 \dots \alpha_N}$ are expressed in terms of the same $A^{1 \dots N}_{\alpha_1 \dots \alpha_N}$ which will be denoted elsewhere by $\Phi_{\alpha_1 \dots \alpha_N}$ This proves [5,6,9] that for compatibility of the overcomplete set of eq (24) it is sufficient that the factorization equations (21) be fulfilled

The last problem we have to attend to is the fulfillment of the periodic boundary conditions (9) Substitution of the expression for the wavefunction in eq (9) and taking eq (24) into account results in the following equations for the isotopic-spin vector $\Phi_{\alpha_1 \dots \alpha_N}$

which also define the eigenvalues of the wavevectors K_n

$$\hat{T}_n \Phi = e^{iK_n L} \Phi \,, \qquad (25)$$

where

$$\hat{T}_n = \hat{S}_{n\,n+1} \ldots \hat{S}_{nN} \hat{S}_{n1} \ldots \hat{S}_{n\,n-1} \qquad (26)$$

The N equations (25) are compatible, i e , $[\hat{T}_n \hat{T}_m] = 0$ by virtue of the factorization equations (21)

To solve eqs (25) let us introduce the operator $\hat{L}(v)$ acting in the 2^{N+1}-dimensional isotopic space

$$\hat{L}(v) = \prod_{n=1}^{N} \hat{S}_{0n}(\tfrac{1}{2}(v - \sigma_n)) \,, \qquad (27)$$

$$\hat{S}_{0n}(\tfrac{1}{2}(v - \sigma_n)) \equiv a(\tfrac{1}{2}(v - \sigma_n)) + b(\tfrac{1}{2}(v - \sigma_n)) P_{0n} \qquad (28)$$

The operator $P_{0n} = \frac{1}{2}(1 + \boldsymbol{\tau}_0 \boldsymbol{\tau}_n)$ interchanges the isotopic indices of the additional particle with the index 0 and the index of the nth particle Let us denote the trace of $\hat{L}(v)$ over the indices of the additional particle by $\hat{T}(v) \equiv \mathrm{Tr}_0 \hat{L}(v)$ The operator $\hat{T}(v)$ acts in the 2^N-dimensional isotopic space and coincides with the particular case of the Baxter transfer matrix [4] As has been shown by Baxter, by virtue of eq (21) we have the relation

$$[\hat{T}(u), \hat{T}(v)] = 0 \qquad (29)$$

It is easy to check that

$$\hat{T}(v = \sigma_n) = -\hat{T}_n \qquad (30)$$

To find the eigenvectors of $\hat{T}(v)$ we shall make use of the beautiful method invented by Faddeev et al [10, 11] This method is based on the property of similarity of the products of two operators $\hat{L}(v)$ and $L(u)$ in different orders which is easily derived from eq (21) Let us write down the operator $\hat{L}(v)$ in a more explicit form

$$\hat{L}(v) \equiv \begin{pmatrix} A(v) & B(v) \\ C(v) & D(v) \end{pmatrix} = \prod_{n=1}^{N} \begin{pmatrix} a_{0n} + b_{0n}\left(\dfrac{1+\tau_n^z}{2}\right) & b_{0n}\tau_n^- \\ b_{0n}\tau_n^+ & a_{0n} + b_{0n}\left(\dfrac{1-\tau_n^z}{2}\right) \end{pmatrix} \qquad (31)$$

Here $a_{0n} \equiv a(\frac{1}{2}(v - \sigma_n))$, $b_{0n} \equiv b(\frac{1}{2}(v - \sigma_n))$ Note that while $\sigma_n = \pm 1$, the parameter v takes on arbitrary values From eqs (21) and (15) it is easy to check the following relations

$$[A(v), B(u)] = \frac{b(u-v)}{a(u-v)} [B(u)A(v) - B(v)A(u)] \,, \qquad (32)$$

$$[D(v), B(u)] = \frac{b(v-u)}{a(v-u)} [B(u)D(v) - B(v)D(u)] \,, \qquad (33)$$

$$[B(u), B(v)] = 0 \qquad (34)$$

It is necessary to diagonalize the operators

$$\hat{T}_n = -[A(\sigma_n) + D(\sigma_n)]$$

It is easy to check that the state Φ_0, with all the spins directed upwards, i e , $\tau_n^z \Phi_0 = \Phi_0$, is an eigenvector of the operator $A(v) + D(v)$

$$[A(v) + D(v)]\,\Phi_0 = \left\{ \prod_{n=1}^{N} [a(\tfrac{1}{2}(v - \sigma_n)) + b(\tfrac{1}{2}(v - \sigma_n))] + \prod_{n=1}^{N} a(\tfrac{1}{2}(v - \sigma_n)) \right\} \Phi_0 \qquad (35)$$

We shall construct the eigenvectors of $A + D$ according to Faddeev et al [11], with the operators $B(v_\alpha)$ acting on Φ_0 as formation operators Relation (32) and (33) remind us of the commutation relations of the formation operator with the hamiltonian The spurious terms occurring in the right-hand sides of eqs (32), (33) due to the second terms are destroyed by the correct set of values of v_α So, the vector Φ should be sought in the form

$$\Phi = \prod_{\alpha=1}^{M} B(v_\alpha)\Phi_0 \qquad (36)$$

Then

$$[A(v) + D(v)]\,\Phi = \Lambda(v, v_1, \ldots, v_M)\Phi \,, \qquad (37)$$

where

$$\Lambda(v, v_1, \ldots, v_M) = \prod_{n=1}^{N} [a(\tfrac{1}{2}(v - \sigma_n)) + b(\tfrac{1}{2}(v - \sigma_n))] \times \prod_{\alpha=1}^{M} \frac{a(\frac{1}{2}(v_\alpha - v)) + b(\frac{1}{2}(v_\alpha - v))}{a(\frac{1}{2}(v_\alpha - v))} \qquad (38)$$

$$+ \prod_{n=1}^{N} a(\tfrac{1}{2}(v - \sigma_n)) \prod_{\alpha=1}^{M} \frac{a(\frac{1}{2}(v - v_\alpha)) + b(\frac{1}{2}(v - v_\alpha))}{a(\frac{1}{2}(v - v_\alpha))}$$

if

$$\prod_{n=1}^{N} \frac{a(\frac{1}{2}(v_\alpha - \sigma_n)) + b(\frac{1}{2}(v_\alpha - \sigma_n))}{a(\frac{1}{2}(v_\alpha - \sigma_n))}$$

$$= -\prod_{\beta=1}^{M} \frac{a(\frac{1}{2}(v_\alpha - v_\beta)) + b(\frac{1}{2}(v_\alpha - v_\beta))}{a(\frac{1}{2}(v_\alpha - v_\beta)) - b(\frac{1}{2}(v_\alpha - v_\beta))} \qquad (39)$$

The last M eqs (39) define the admissible values of v_α and are a consequence of the requirement of annihilation of the spurious terms in the right-hand side of eq (37) The sufficiency of these conditions was proven in ref [10] The necessity becomes evident from the following statement belonging to S Manakov The left-hand side of eq (37) has no poles at $v = v_\alpha$ Therefore, eqs (39) arise as the equations of the annihilation of the residues in the points where $a(\frac{1}{2}(v - v_\alpha))$ becomes zero, since it is evident from eq (15) that $a(0) = 0$ Substituting the explicit expressions for $a(v)$ and $b(v)$ into eqs (38) and (39) and introducing instead of v_α the quantities q_α related as, $q_\alpha = v_\alpha + i\lambda$, we obtain

$$e^{iK_nL} = -\prod_{l=1}^{N} \frac{\sigma_n - \sigma_l + 2i\lambda_1}{\sigma_n - \sigma_l - 2i\lambda_1} \prod_{\alpha=1}^{M} \frac{q_\alpha - \sigma_n + i\lambda}{q_\alpha - \sigma_n - i\lambda}, \qquad (40)$$

$$\prod_{n=1}^{N} \frac{q_\alpha - \sigma_n + i\lambda}{q_\alpha - \sigma_n - i\lambda} = -\prod_{\beta=1}^{M} \frac{q_\alpha - q_\beta + 2i\lambda}{q_\alpha - q_\beta - 2i\lambda} \qquad (41)$$

Here

$$\lambda = (\lambda_1 - \lambda_0)/(1 + \lambda_1 \lambda_0) = \operatorname{tg} 2g_v$$

Eqs (40) and (41) should be supplemented by the relations for the energy and the momentum of the given state

$$P = \sum_{n=1}^{N} K_n, \qquad (42)$$

$$E = \sum_{n=1}^{N} \sigma_n K_n \qquad (43)$$

Eqs (40)–(43) completely describe the eigenstates of hamiltonian (2)

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