Lecture 7. ϕ^4 theory.

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1. Wick's theorem in KG theory.

As we have seen from previous lectures there is the functional integral representation for the expectation value of time-ordered Heisenberg operators (Green's functions)

$$< 0|T(\hat{\phi}(\vec{x}_N,t_N)...\hat{\phi}(\vec{x}_1,t_1)|0 >= \lim_{T\to\infty}\frac{\int[D\phi]\phi(\vec{x}_1,t_1)...\phi(\vec{x}_N,t_N)\exp\left[\frac{\imath}{\hbar}\int_{-T}^{T}dtd^3x\mathcal{L}(\phi,\partial\phi)\right]}{\int[D\phi]\exp\left[\frac{\imath}{\hbar}\int_{-T}^{T}dtd^3x\mathcal{L}(\phi,\partial\phi)\right]} \tag{1}$$

This representation in turn, has been obtained as a result of certain continuation of the functional integral representation for the correlation function determined in euclidean space

$$< \phi(x_1)...\phi(x_N) >= \frac{1}{Z}\int[D\phi]\phi(x_1)...\phi(x_N)\exp\left(-A[\phi]\right) \tag{2}$$

where the euclidean KG theory action $A[\phi])$

$$A[\phi] = \int d^4x(\frac{1}{2}(\partial_a\phi)^2 + \frac{m^2}{2}\phi^2) \tag{3}$$

is obtained from KG action in Minkowski space-time by the substitution $t = -\imath x_4$ and

$$Z = \int[D\phi]\exp\left(-A[\phi]\right) \tag{4}$$

Wick's theorem allows to calculate the correlation functions above in QKG theory using the formal properties of the functional integral.

1.1. THE THEOREM.

1. The correlation function of odd number of KG fields vanish:

$$< \phi(x_1)...\phi(x_{2N+1}) >= 0 \tag{5}$$

The statement follows from the fact that action A is even under the change $\phi \to -\phi$ but the integrand is odd.

2. 2-point correlation function is

$$< \phi(x_1)\phi(x_2) >= D(x_1 - x_2),\ \ D(x) = \int \frac{d^4p}{(2\pi)^4}\frac{\exp(\imath px)}{p^2+m^2} \tag{6}$$

(euclidean space propagator).

3. $2N$**-point correlation function is a sum of** $(2N-1)!! \equiv 135...(2N-1)$ **terms, such that each term corresponds to one of** $(2N-1)!!$ **pairings between the fields** $\phi(x_1)...\phi(x_{2N})$ **and the paired fields produce a factor**

$$D(x_1 - x_2) \tag{7}$$

This pairing rule can be expressed as a reccurent relation

$$< \phi(x_1)...\phi(x_{2N}) >= \sum_{j=1}^{2N} D(x_1 - x_j) < \phi(x_2)...\check{\phi}(x_j)...\phi(x_{2N}) > \tag{8}$$

where the field $\phi(x_j)$ is missing. Thus, to prove the theorem it sufficies to derive this reccurrent relation.

1.2. PROOF OF THE THEOREM BY SCHWINGER-DYSON EQUATION.

Consider the change of variables

$$\phi(x) \to \tilde{\phi}(x) = \phi(x) + \epsilon(x) \tag{9}$$

in the above path integral defining $2N-1$ correlation function:

$$\frac{1}{Z}\int [D\tilde{\phi}]\tilde{\phi}(x_1)...\tilde{\phi}(x_{2N-1})\exp(-A[\tilde{\phi}]) \tag{10}$$

where $\epsilon(x)$ is an arbitrary infinitesimal function going to zero as $x \to \infty$. Then

$$A[\phi+\epsilon] = A[\phi] + \int d^4x\epsilon(x)(m^2 - \partial_a^2)\phi(x) + ... \tag{11}$$

The value of the path integral does not change under the change of variables, moreover, the mesure $[D\phi]$ is invariant under the change (9) and hence

$$0 = \sum_{j=1}^{2N-1} \epsilon(x_j) \int [D\phi]\phi(x_1)...\check{\phi}(x_j)...\phi(x_{2N-1}) \exp{(-A[\phi])} -$$
$$\int d^4x\epsilon(x)(m^2 - \partial_a^2) \int [D\phi]\phi(x)\phi(x_1)...\phi(x_{2N-1}) \exp{(-A[\phi])} + ... \tag{12}$$

Because of $\epsilon(x)$ is an arbitrary this equation is equivalent to

$$(m^2 - (\frac{\partial}{\partial x^a})^2) < \phi(x)\phi(x_1)...\phi(x_{2N-1}) >=$$
$$\sum_{j=1}^{2N-1} \delta^4(x - x_j) < \phi(x_1)...\check{\phi}(x_j)...\phi(x_{2N-1}) > \tag{13}$$

This equation, known as Schwinger-Dyson equation is a quantum version of the classical equation of motion in KG theory. Indeed, when $x \neq x_j$ the field $\phi(x)$ obeys KG equation of motion.

The solution of this equation is expressed by the function D which is determined by

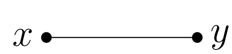
$$(m^2 - \partial_a^2)D(x - y) = \delta^4(x - y) \tag{14}$$

and is given by (6). We have thus proved the recurrent relation (8).

1.3. DIAGRAMS.

The Wick's theorem rule can be represented by diagrams.

The function $D(x - y)$ is represented by a line connecting x and y

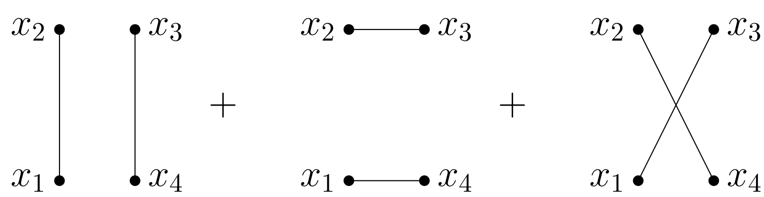
$D(x - y) =$ 

(15)

Then applying the recurrent formula we obtain a sum of terms with all possible pairings between the fields which can be represented as a set of lines.

Let us consider for example the result of 4-point function calculation

$$< \phi(x_1)...\phi(x_4) >=$$



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2. Correlation functions Path Integral representation and Schwinger-Dyson equation for ϕ^4 field theory model.

2.1 THE EUCLIDEAN ACTION AND CORRELATION FUNCTIONS PATH INTEGRAL REPRESENTATION.

The **euclidean** action is given by

$$A[\phi] = \int d^4x(\frac{1}{2}(\partial_a\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4) \tag{17}$$

It is not a free field theory due to the self-interaction term $\frac{\lambda}{4!}\phi^4$ which is characterized by the coupling constant λ.

We consider first the theory in euclidean space, where we are interested in the correlation functions. As we have discussed in the previous lectures the correlation functions are given by the functional integral:

$$< \phi(x_1)...\phi(x_N) >= \frac{1}{Z}\int [D\phi]\phi(x_1)...\phi(x_N)\exp(-A[\phi]) \tag{18}$$

This functional integral representation is used to get the expectation values of (real) time-ordered Heisenberg operators in Minkowski space-time

(Green's functions) by the continuation from the complex values x_i^4 to the limiting values $x_i^4 = \imath t_i$.

The action is not quadratic in ϕ so the functional integral can not be evaluated in an explicit form.

2.2 SCHWINGER-DYSON EQUATION.

Similar to the KG theory, one can deduce certain equations for the correlation fuctions using the invariance of $[D\phi]$ under the shift

$$\phi(x) \to \phi(x) + \epsilon(x) \tag{19}$$

with arbitrary function $\epsilon(x)$. **This is an QFT analog of minimal action principal in CFT.**

Consider the change of variables

$$\phi(x) \to \tilde{\phi}(x) = \phi(x) + \epsilon(x) \tag{20}$$

in the above path integral defining N-points correlation function:

$$\frac{1}{Z}\int [D\tilde{\phi}]\tilde{\phi}(x_1)...\tilde{\phi}(x_N)\exp\left(-A[\tilde{\phi}]\right) \tag{21}$$

where $\epsilon(x)$ is an arbitrary infinitesimal function going to zero as $x \to \infty$. Then

$$A[\phi+\epsilon] = A[\phi] + \int d^4x\epsilon(x)(m^2\phi - \partial_a^2\phi + \frac{\lambda}{3!}\phi^3)(x) + ... \tag{22}$$

The value of the path integral does not change under the change of variable and the mesure $[D\phi]$ is invariant under the change (20), hence

$$0 = \sum_{j=1}^{N} \epsilon(x_j) \int [D\phi] \phi(x_1)...\check{\phi}(x_j)...\phi(x_N) \exp(-A[\phi]) -$$
$$\int [D\phi] \int d^4x \epsilon(x)(m^2\phi(x) - \partial_a^2 \phi(x) + \frac{\lambda}{3!}\phi^3(x))\phi(x_1)...\phi(x_N) \exp(-A[\phi]) \tag{23}$$

Because of $\epsilon(x)$ is an arbitrary this equation is equivalent to

$$< (m^2\phi(x) - (\frac{\partial}{\partial x^a})^2\phi(x) + \frac{\lambda}{3!}\phi^3(x))\phi(x_1)...\phi(x_N) >=$$
$$\sum_{j=1}^{N} \delta^4(x - x_j) < \phi(x_1)...\check{\phi}(x_j)...\phi(x_N) > \tag{24}$$

This equation (Schwinger-Dyson equation) is a quantum version of the classical equation of motion in ϕ^4 theory. Indeed, the Euler-Lagrange equation in the classical ϕ^4 theory is

$$m^2\phi(x) - (\frac{\partial}{\partial x^a})^2\phi(x) + \frac{\lambda}{3!}\phi^3(x) = 0 \tag{25}$$

The Schwinger-Dyson equation means that **field insertion $\phi(x)$ in any correlation function satisfies the same equation in the week sense. In other words, it is satisfied everywhere except the points $x_1, ...x_N$.**

Unlike the KG theory, SD equation for interacting ϕ^4 theory **is not a closed equation for correlator** $< \phi(x)... >$ and can not be solved for the correlator because it involves the correlation function

$$< \phi^3(x)... > \tag{26}$$

which has no simple expression in terms of $< \phi(x)... >$.

3. Perturbation theory and Feynman Diagrams.

3.1. PERTURBATION THEORY.

The perturbation theory is the practical way to calculate the correlation functions in an interacting theory. The idea is to write

$$A = A_0 + A_I \tag{27}$$

such that functional integral with A_0 alone can be evaluated explicitely and expand in A_I:

$$\exp(-A_0 - A_I) = \exp(-A_0)(1 - A_I + \frac{1}{2}A_I^2 - ...) \tag{28}$$

In our case

$$A_0 = A_{KG} \ , \ A_I = \frac{\lambda}{4!}\int d^4x\phi^4(x) \tag{29}$$

Thus, the perturbation expansion is an expansion in the coupling constant λ.

3.2. 2-POINT CORRELATION FUNCTION AND WICK'S.

Let us apply the perturbation expansion to the 2-point correlation function

$$\begin{aligned} < \phi(x_1)\phi(x_2) >= \frac{1}{Z}\int [D\phi]\phi(x_1)\phi(x_2)\exp(-A_0)\exp(-\frac{\lambda}{4!}\int d^4x\phi^4(x)) = \\ \frac{1}{Z}\int [D\phi]\phi(x_1)\phi(x_2)\exp(-A_0)(1 - \frac{\lambda}{4!}\int d^4x\phi^4(x) + ...) \end{aligned} \tag{30}$$

We see that zero-order term is the correaltion function for the KG theory so that we have a series

$$< \phi(x_1)\phi(x_2) >=< \phi(x_1)\phi(x_2) >_0 -\frac{\lambda}{4!}\int d^4x < \phi(x_1)\phi(x_2)\phi^4(x) >_0 +... \tag{31}$$

where $< ... >_0$ means the correlation function of the KG theory and the factor $1/Z$ is omitted for a moment. This is the general situation:

the perturbation theory represents the correlation function of interacting theory as a power series whose coefficients are certain integrals of correlation functions of the unperturbed theory.

The correlation functions of unperturbed theory are calculated by the Wick's rule. For example, in the first order the contribution is given by the integral of 6-point correlation function of KG theory

$$< \phi(x_1)\phi(x_2)\phi^4(x) >_0=< \phi(x_1)\phi(x_2)\phi(x)\phi(x)\phi(x)\phi(x)\phi(x)\phi(x) >_0 \tag{32}$$

According to the Wick's rule this correaltion function is given by a sum of products of propagators corresponding to all possible pairings.

3.3. TYPES OF PAIRINGS AND DIAGRAMS.

There are two different pairings here:

1. We can contract $\phi(x_1)\phi(x_2)$ and then pair the fields $\phi(x)\phi(x)\phi(x)\phi(x)$ to each other by 3 different ways:

$$< \phi(x_1)\phi(x_2)\phi(x)\phi(x)\phi(x)\phi(x) > + < \phi(x_1)\phi(x_2)\phi(x)\phi(x)\phi(x)\phi(x) > +... \tag{33}$$

As a result we obtain

$$3D(x_1 - x_2)D(x - x)D(x - x) \tag{34}$$

2. One can contract $\phi(x_1)$ with one of $\phi(x)$, contract $\phi(x_2)$ with another $\phi(x)$ and then contract to each other the remaining $\phi(x)$:

$$< \phi(x_1)\phi(x_2)\phi(x)\phi(x)\phi(x)\phi(x) > \tag{35}$$

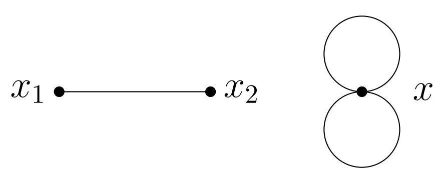
We obtain

$$12D(x_1 - x)D(x_2 - x)D(x - x) \tag{36}$$

The resulting contribution is

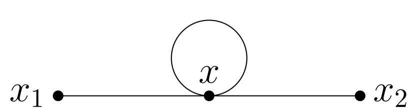
$$-\frac{\lambda}{4!}[3D(x_1 - x_2)\int d^4xD(x-x)^2 + 12\int d^4xD(x_1 - x)D(x_2 - x)D(x - x)] \tag{37}$$

These terms can be represented by the following diagrams: 3 diagrams of type



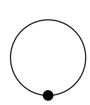
(38)

plus 12 diagrams of the type



(39)

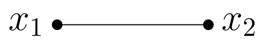
where

 $= D(x - x)$

(40)

The main elements in these diagrams are:

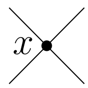
1. The lines (propagators)



$$= D(x_1 - x_2) \tag{41}$$

showing how the paticle propagates, and

2. The interaction vertices:



$$= -\lambda \int d^4x, \tag{42}$$

the points where 4 propagators meet and the point x associated with the vertex is integrated out.

Though the last 12 diagrams describe the process of self-interaction during the particle is propagating from the point x_1 to the point x_2 the first 3 diagrams contain the factor describing the process of creation and annihilation of particles from vacuum in the point x which is not affecting the propagation of a particle from x_1 to x_2. This is an example of so called **vacuum diagram**. Moreover the diagram (38) is an example of **disconnected diagram** describing two processes that do not affect each other.

3.4. SECOND ORDER DIAGRAMS AND SYMMETRY FACTORS.

Second order in λ contribution is given by

$$\frac{1}{2!}(-\frac{\lambda}{4!})^2 < \phi(x_1)\phi(x_2) \int d^4x\phi(x)\phi(x)\phi(x)\phi(x) \int d^4y\phi(y)\phi(y)\phi(y)\phi(y) > \tag{43}$$

where again Z^{-1} is ignored. In orter to calculate this one has to consider all possible ways of contracting all the fields operators inside this correlation function.

Let us consider an example of the contribution which is given by all possible contractions of the fields $\phi(x_1)$ and $\phi(x_2)$ with the fields from the first integration, contractions of the remaining couple of fields from the first integration with the fields from the second integration as well as the contraction of the remaining pair of fields from the second integration with themselves:

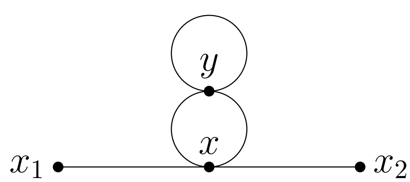
$$\frac{1}{2!}(-\frac{\lambda}{4!})^2 \int d^4xd^4y < \phi(x_1)\phi(x_2)\phi(x)\phi(x)\phi(x)\phi(x)\phi(y)\phi(y)\phi(y)\phi(y) > \tag{44}$$

The total number of contractions of this type is equal

$$(2 \times 4) \cdot 3 \cdot 4 \cdot 3 \cdot 1 = 288 = (12)^2 \cdot 2 \tag{45}$$

Indeed, we have $(4 \cdot 3)(4 \cdot 3)$ ways to contract a pair of fields $\phi(x)$ with a pair of fields $\phi(y)$. For each of these variants there are two ways of pairing the fields $\phi(x_1)\phi(x_2)$ with the rest pair of the fields $\phi(x)$ and the only contraction for the pair of fields $\phi(y)$.

The diagram associated with this contractions is



(46)

By our rules, each propagator is D, and each vertex is $-\lambda \int d^4x$, hence the contribution is

$$(-\lambda)^2 \int d^4xD(x_1 - x)D(x_2 - x) \int d^4yD(x - y)D(x - y)D(y - y) \tag{47}$$

This contribution comes with the factor

$$\frac{2(12)^2}{2!(4!)^2} = \frac{1}{4} \tag{48}$$

Thus the nominator and denominator factors nearely cancell each other.

The n-th order diagrams comes with the factor

$$\frac{(-\lambda)^n}{n!(4!)^n} \tag{49}$$

But now the actual contribution involves integrations like

$$\int d^4y_1\phi^4(y_1)...\int d^4y_n\phi^4(y_n) \tag{50}$$

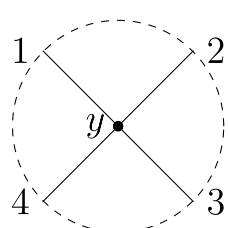
Therefore, any contruction gives a combinatoric factor $n!$ due to the possibility to permute these integrations. Next, each integration

$$\int d^4y\phi(y)\phi(y)\phi(y)\phi(y) \tag{51}$$

is typically contracted with 4 distinct external fields $\phi(x)$:

$$(\phi(y)\phi(y)\phi(y)\phi(y))\phi_1\phi_2\phi_3\phi_4 \tag{52}$$

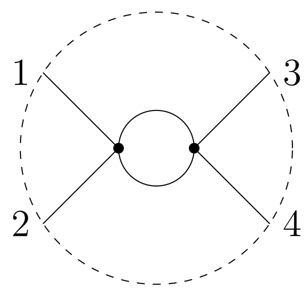
It corresponds to the following diagram fragment



(53)

The possibility to permute the fields $\phi(y)$ typically yields the factor 4! for such vertex. That is why we associate the vertex the factor $(-\lambda)$ instead of $(-\frac{\lambda}{4!})$.

The above calculation of multiplicity of each contraction **may overestimate** the actual multiplicity of diagram. For example, if two ϕ in one vertex are contracted with two ϕ in another one, as in the diagram fragment



$$\tag{54}$$

$$\phi_1\phi_2(\phi\phi\phi\phi)(\phi\phi\phi\phi)\phi_3\phi_4 \tag{55}$$

we obtain the factor

$$(4 \times 3)^2 \cdot 2 = \frac{(4!)^2}{2} \tag{56}$$

not $(4!)^2$. The additional factor $\frac{1}{2}$ here is called a **symmetry factor**, as it reflects the symmetry of the above fiagram fragment w.r.t. the permutation of two propagators. In general, each diagram carries the symmetry factor

$$\frac{1}{S}\ , \ \textit{where S is the order of diagrams symmetry group} \tag{57}$$

The factor 1/4 we observed in the diagram (46) is just this symmetry factor, as we can permute two propagators from the point x to y and permute also two ends in the propagator from y to y. Also we saw that the first order diagrams (38) and (39) come with the factors 1/8 and 1/2.

In the symmetry factor 1/8 the first and second 1/2 appear because the contractions

$$\overbrace{\phi(x)\phi}(x) \tag{58}$$

are symmetric w.r.t. permutations of the ends of contraction. The third 1/2 comes from the permutations of the two contractions:

$$\overbrace{\phi(x)\phi}(x) \leftrightarrow \overbrace{\phi(x)\phi}(x) \tag{59}$$

3.5. FEYNMAN RULES.

Now we can formulate the rules of diagramatics, or **Feynman rules**, to find the actual contribution of a diagram.

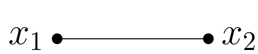
To find λ^n contribution to

$$< \phi(x_1)...\phi(x_N) > \tag{60}$$

1. Draw all diagrams with n vertices and N external points x_1,...,x_N.

2. For each diagram associate:

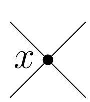
A. the propagator line



$$= D(x_1 - x_2) \tag{61}$$

for each propagator.

B. the vertex diagram



$$= -\lambda \int d^4x \tag{62}$$

for each interaction vertex.

C. Multiply the diagram by the symmetry factor $1/S$.

3. Sum over the values of all diagrams.

3.6. FEYNMAN RULES IN MINKOWSKI SPACE-TIME.

The Feynman rules above have been obtained in euclidean space. As we know, one can continue the functional integral definition of correlation functions to the complex values $(x_i)^4 = \tau_i$ and specialize it to $x_i^4 = \imath t_i$. In this situation, we obtain functional representation for the Green's functions or the time-ordered expectation values in Minkowski space-time. Thus, **the perturbation theory we have applied for the correlation functions calculation of ϕ^4 theory in euclidean space after the continuation gives the perturbation theory in Minkowski space-time**. It leads to the Feynman rules for the calculation of Green's functions.

To find these rules we write first the action of ϕ^4-theory in Minkowski space:

$$S[\phi] = \int d^4x(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4) \tag{63}$$

One can again expand the action into the sum of KG action and interaction term

$$S = S_0 + S_I \ , \ S_0 = \int d^4x(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2) \ , \ S_I = -\frac{\lambda}{4!}\int d^4x\phi^4(x) \tag{64}$$

The continuation into Minkowski space gives the expansion:

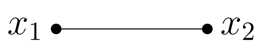
$$\exp(\imath S_0 + \imath S_I) = \exp(\imath S_0)(1 + \imath S_I + \frac{\imath^2}{2!}S_I^2 ...) \tag{65}$$

as well as the expansion for the 2-points Green's function

$$
\begin{aligned}
&< T(\phi(x_1)\phi(x_2)) >= \\
&\frac{1}{Z}\int [D\phi]\phi(x_1)\phi(x_2)\exp{(\imath S_0)}\exp{(-\imath\frac{\lambda}{4!}\int d^4x\phi^4(x))} = \\
&\frac{1}{Z}(\int [D\phi]\phi(x_1)\phi(x_2)\exp{(\imath S_0)} - \imath\frac{\lambda}{4!}\int [D\phi]\phi(x_1)\phi(x_2)\int d^4x\phi^4(x)\exp{(\imath S_0)} + ...)
\end{aligned}
\tag{66}
$$

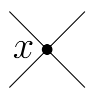
It leads to the following Feynman rules in Minkowski space:

A. For each propagator



$$= D_F(x_1 - x_2) \tag{67}$$

B. For each vertex



$$= (-\imath\lambda)\int d^4x \tag{68}$$

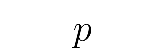
C. Multiply the diagram by the symmetry factor $1/S$.

3.7. FEYNMAN RULES IN MOMENTA REPRESENTATION.

It is convenient also to represent the Feynman rules in momentum space taking the Fourier transformation for each Feynman propagator D_F

$$D_F(x-y) = \int \frac{d^4p}{(2\pi)^4}\frac{\imath}{p^2 - m^2 + \imath\epsilon}\exp{(-\imath p(x-y))} \tag{69}$$

Thus, in momenta space this propagator can be represented by a line with an arrow and momentum p indicated:



$$= \frac{\imath}{p^2 - m^2 + \imath\epsilon} \tag{70}$$

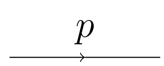
Then if 4 lines meet at interaction vertex at x the integration over x gives

$$\int d^4x \exp(-\imath p_1 x) \exp(-\imath p_2 x) \exp(-\imath p_3 x) \exp(-\imath p_4 x) = (2\pi)^4 \delta^4(p_1 + ... + p_4) \tag{71}$$

where all momenta p_i inflow to the point x. It means that total momentum is conserved at the point.

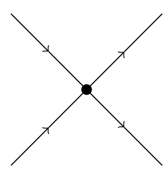
It gives the following Feynman rules in momenta representation:

A. For each propagator



$$= \frac{\imath}{p^2 - m^2 + \imath\epsilon} \tag{72}$$

B. For each vertex



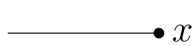
$$= (-\imath\lambda) \tag{73}$$

3.8. GENERALIZATION OF ϕ^4 THEORY.

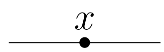
The strightforward generalization of scalar ϕ^4 theory is to consider the following interaction functional

$$A_I = \int d^4x(\lambda_1(x)\phi(x) + \frac{\lambda_2(x)}{2!}\phi^2(x) + \frac{\lambda_3(x)}{3!}\phi^3(x) + \frac{\lambda_4(x)}{4!}\phi^4(x) + ...) \tag{74}$$

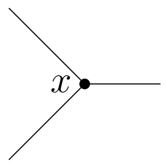
Here the coupling constants λ_i are taken to be arbitrary (fixed) functions of x. interaction includes the linear and quadratic terms in ϕ, although they can be attributed to the free action A_0. The generalization of the above Feynman rules amounts to including new vertices



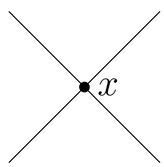
$$= -\int d^4x \lambda_1(x) \tag{75}$$



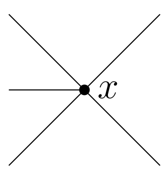
$$= -\int d^4x \lambda_2(x) \tag{76}$$



$$= -\int d^4x \lambda_3(x) \tag{77}$$



$$= -\int d^4x \lambda_4(x) \tag{78}$$



$$= -\int d^4x \lambda_5(x) \tag{79}$$

... ...

Allowing coupling constants to be a functions is often just a usefull technical trick and in most of the practical applications one deals with the **homogeneous** interaction.