Lecture 7.

Electroweak and Standard models.

Plan.

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3.1. Gauge group and bosonic sector.

3.2. Quarks multiplets and Lagrangian.

Appendix.

1. QED+ complex scalar, chirality and Lorentz group (recalling).

1.1. The QED + complex scalar Lagrangian.

$$
\begin{aligned}
&L = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu\Phi|^2 - V(\Phi) + \\
&\bar{\psi}_L(\imath\gamma^\mu D_\mu)\psi_L + \bar{\psi}_R(\imath\gamma^\mu\partial_\mu)\psi_R - \lambda_f(\bar{\psi}_L\Phi\psi_R + \bar{\psi}_R\Phi^*\psi_L), \\
&D_\mu = \partial_\mu + \imath e A_\mu, \\
&V(\Phi) = -\mu^2\Phi^*\Phi + \frac{\lambda}{2}(\Phi^*\Phi)^2, \ \Phi = \frac{1}{\sqrt{2}}(\Phi^1 + \imath\Phi^2)
\end{aligned}
\tag{1}
$$

Recall what are ψ_L, ψ_R and haw they transform under the Lorentz transformations.

The Clifford algebra is generated by the identity matrix and the gamma matrices γ^μ, $\mu = 0, ..., 3$ satisfying

$$
\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{2}
$$

Let us consider some special element of algebra

$$\gamma^5 \equiv \imath\gamma^0...\gamma^3,\ (\gamma^5)^\dagger = \gamma^5$$
$$\{\gamma^5, \gamma^\mu\} = 0,$$
$$(\gamma^5)^2 = 1 \tag{3}$$

Then it follows that

$$P_L \equiv \frac{1-\gamma^5}{2},\ P_R \equiv \frac{1+\gamma^5}{2} \tag{4}$$

are the orthogonal projection operators:

$$P_L^2 = P_L,\ P_R^2 = P_R,\ P_L P_R = 0,\ P_L + P_R = 1 \tag{5}$$

Hence the Dirac spinors space, which is Clifford algebra representation can be decomposed into the direct sum of vector spaces:

$$\psi = (P_L + P_R)\psi \equiv \psi_L + \psi_R,$$
$$\gamma^5\psi_L = -\psi_L,\ \gamma^5\psi_R = \psi_R \tag{6}$$

The spinors ψ_L, ψ_R are called Weyl's spinors.

They are important because the spaces of Weyl spinors form (irreducible) representations of Lorentz algebra. Indeed, the Loerntz algebra generators commute to the projection operators:

$$S^{\mu\nu} \equiv \frac{\imath}{4}[\gamma^\mu, \gamma^\nu],$$
$$[S^{\mu\nu}, P_{L,R}] = 0 \Leftrightarrow$$
$$\gamma^5 S^{\mu\nu}\psi_L = S^{\mu\nu}\gamma^5\psi_L = -S^{\mu\nu}\psi_L,$$
$$\gamma^5 S^{\mu\nu}\psi_R = S^{\mu\nu}\gamma^5\psi_R = S^{\mu\nu}\psi_R \tag{7}$$

Hence, **the Lagrangian (1) is Lorentz invariant.**

Gauge transformations are given by

$$
\begin{aligned}
&\delta A_\mu = -\frac{1}{e}\partial_\mu \alpha \\
&\Phi(x) \to \exp\left(\imath\alpha(x)\right)\Phi(x), \\
&\psi_L \to \exp\left(\imath\alpha(x)\right)\psi_L, \;\; \psi_R \to \psi_R
\end{aligned} \tag{8}
$$

so **the Lagrangian (1) is also gauge invariant.**

Let us also check that the covariant derivative terms are consistent with Weyl fermions constraints:

$$
\begin{aligned}
&\bar{\psi}_R \imath\gamma^\mu D_\mu \psi_L = \psi^\dagger \frac{1+\gamma^5}{2}\gamma^0 \imath\gamma^\mu D_\mu \psi_L = \psi^\dagger \gamma^0 \frac{1-\gamma^5}{2}\gamma^0 \imath\gamma^\mu D_\mu \psi_L = \\
&\bar{\psi}\imath\gamma^\mu D_\mu \frac{1+\gamma^5}{2}\psi_L = 0
\end{aligned} \tag{9}
$$

But **the Lagrangian (1) is not invariant under the change $\psi_L \leftrightarrow \psi_R$.** It means that the symmetry under the spacial reflection

$$
\begin{aligned}
&P : (x^0, \vec{x}) \to (x^0, -\vec{x}), \\
&P : \psi(x) \to \gamma^0 \psi(Px)
\end{aligned} \tag{10}
$$

is broken for the Lagrangian (1). By this reason it is impossible to add standard massive term for fermions

$$
m\bar{\psi}\psi = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) \tag{11}
$$

so as not to destroy the $\psi_L \leftrightarrow \psi_R$ assymetric gauge interraction.

Instead, **the fermions become massive by the Higgs mechanism**.

This left-right (P) asymmetry is similar to that in the GSW model of electro-weak interractions. The fermions interract with the dublet of complex scalar fields in such a way to conserve the gauge invariance and get the masses by the Higgs mechanism:

$$\Phi(x) = \frac{1}{\sqrt{2}}(v + h(x) + \imath\phi(x)),$$
$$\lambda_f(\bar{\psi}_L\Phi\psi_R + \bar{\psi}_R\Phi^*\psi_L) =$$
$$\lambda_f(\Phi\bar{\psi}\frac{1+\gamma^5}{2}\psi + \Phi^*\bar{\psi}\frac{1-\gamma^5}{2}\psi) =$$
$$\frac{\lambda_f}{\sqrt{2}}((v + h(x) + \imath\phi(x))\bar{\psi}\frac{1+\gamma^5}{2}\psi + (v + h(x) - \imath\phi(x))\bar{\psi}\frac{1-\gamma^5}{2}\psi) =$$
$$m_f\bar{\psi}\psi + \frac{\lambda_f}{\sqrt{2}}(h(x)\bar{\psi}\psi + \imath\phi(x)\bar{\psi}\gamma^5\psi),$$
$$m_f = \frac{\lambda_f v}{\sqrt{2}} \tag{12}$$

2. Electro-weak Theory (GWS).

Electro-weak interractions theory describes weak and electromagnetic interraction in a unified way. At the same time it is a theory with spontaneously broken symmetry.

2.1. Gauge group, bosonic sector and Higgs effect (recalling).

It is given by YM field theory with gauge group $SU(2) \times U(1)$ interracting with a dublet of complex scalar fields $\Phi(x) = (\phi^1(x), \phi^2(x))$ with the following rule of gauge transformations

$$\Phi(x) \to \exp{(\imath\alpha^a(x)\frac{\sigma^a}{2} + \imath\frac{\beta(x)}{2})}\Phi(x),$$
$$A_\nu(x) \equiv A^a_\nu(x)\frac{\sigma^a}{2} \to U(x)A_\nu(x)U^{-1}(x) + \frac{\imath}{g}U(x)\partial_\nu U^{-1}(x),$$
$$B_\nu(x) \to B_\nu(x) + \frac{\imath}{2\acute{g}}\partial\beta(x) \tag{13}$$

where $U(x) = \exp{(\imath\alpha^a(x)\frac{\sigma^a}{2})}$.

The corresponding part of the Lagrangian is given by

$$L(A,B,\Phi) = -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}|D_\mu\Phi|^2,$$
$$D_\mu\Phi = (\partial_\mu - \imath g A^a_\mu \frac{\sigma^a}{2} - \imath\frac{\acute{g}}{2}B_\mu)\Phi \qquad (14)$$

It is supposed that Φ acquires the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}}(0,v) \qquad (15)$$

due to the self-interraction

$$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 \qquad (16)$$

so that the subgroup of matrices leaves the vacuum vector fixed:

$$\exp\left(\imath\beta(x)(\frac{\sigma^3+1}{2})\right)\Phi_0 = \Phi_0,$$
$$\exp\left(\imath\beta(x)(\frac{\sigma^3+1}{2})\right) \in U(1)_{em} \subset U(1)\times U(1) \subset SU(2)\times U(1) \qquad (17)$$

isomorphic to $U(1)$ group. Therefore we have a massless gauge boson while 3 other bosons becomes massive. Indeed

$$D_\mu(\Phi_0+\phi(x)) = (\partial_\mu - \imath g A^a_\mu\frac{\sigma^a}{2} - \imath\frac{\acute{g}}{2}B_\mu)(\Phi_0+\phi(x)) =$$
$$D_\mu\phi(x) - (\imath g A^a_\mu\frac{\sigma^a}{2} + \imath\frac{\acute{g}}{2}B_\mu)\Phi_0,$$
$$\Phi_0^\dagger(gA^a_\mu\frac{\sigma^a}{2} + \frac{\acute{g}}{2}B_\mu)(gA^{a\mu}\frac{\sigma^a}{2} + \frac{\acute{g}}{2}B^\mu)\Phi_0 =$$
$$\frac{1}{2}\frac{v^2}{4}(g^2(A^1_\mu)^2 + g^2(A^2_\mu)^2 + (-gA^3_\mu + \acute{g}B_\mu)^2) \qquad (18)$$

It make sense to introduce the following combinations of gauge bosons

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1 \mp \imath A^2)_\mu,\ m_W = \frac{gv}{2},$$
$$Z_\mu = \frac{1}{\sqrt{g^2+\acute{g}^2}}(gA^3 - \acute{g}B)_\mu,\ m_Z = \sqrt{g^2+\acute{g}^2}\frac{v}{2}$$
$$A_\mu = \frac{1}{\sqrt{g^2+\acute{g}^2}}(\acute{g}A^3 + gB)_\mu,\ m_A = 0 \qquad (19)$$

For the case of general representation of the gauge group $SU(2) \times U(1)$

$$
\begin{aligned}
& D_\mu = \partial_\mu - \imath g A^a_\mu T^a - \imath \acute{g} Y B_\mu = \\
& \partial_\mu - \imath \frac{g}{\sqrt{2}} (W^+ T^+ + W^- T^-)_\mu - \frac{\imath}{\sqrt{g^2 + \acute{g}^2}} Z_\mu (g^2 T^3 - \acute{g}^2 Y) \\
& - \frac{\imath g \acute{g}}{\sqrt{g^2 + \acute{g}^2}} A_\mu (T^3 + Y)
\end{aligned} \tag{20}
$$

where $T^\pm = T^1 \pm \imath T^2$.

It is natural to identify the EM gauge potential coupling to the charge of electron

$$
e = \frac{g \acute{g}}{\sqrt{g^2 + \acute{g}^2}} \tag{21}
$$

and determine the electric charge operator as

$$
Q_{em} = T^3 + Y \tag{22}
$$

It is also convenient to introduce the mixing angle Θ_W by the relation of two basic fields

$$
\begin{aligned}
& \begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} cos\Theta_W & -sin\Theta_W \\ sin\Theta_W & cos\Theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix} \Leftrightarrow \\
& cos\Theta_W = \frac{g}{\sqrt{g^2 + \acute{g}^2}}, \; sin\Theta_W = \frac{\acute{g}}{\sqrt{g^2 + \acute{g}^2}}
\end{aligned} \tag{23}
$$

Then we will have

$$
\begin{aligned}
& D_\mu = \\
& \partial_\mu - \imath \frac{g}{\sqrt{2}} (W^+ T^+ + W^- T^-)_\mu - \imath \frac{g}{cos\Theta_W} Z_\mu (T^3 - sin^2\Theta_W Q_{em}) - \imath e A_\mu Q_{em}, \\
& g = \frac{e}{sin\Theta_W}
\end{aligned} \tag{24}
$$

and $m_W = m_Z cos\Theta_W$. Experimental data: $m_W = 80 Gev$, $m_Z = 91 Gev$, $m_H = 126 Gev$ (2012).

2.2. Fermionic sector, leptons multiplets.

The leptons (which are fermions) interract to W-bosons only by the left-handed components while the right-handed components do not interract to W. Thus, the left-handed components sit at $SU(2)$ dublets:

$$\begin{pmatrix} \nu_e(x) \\ e^-(x) \end{pmatrix}_L, \begin{pmatrix} \nu_\mu(x) \\ \mu^-(x) \end{pmatrix}_L, \begin{pmatrix} \nu_\tau(x) \\ \tau^-(x) \end{pmatrix}_L \tag{25}$$

Each component of the each dublet is a left-handed Weyl spinor w.r.t. Lorentz group:

$$\gamma^5 \begin{pmatrix} \nu_e(x) \\ e^-(x) \end{pmatrix}_L = - \begin{pmatrix} \nu_e(x) \\ e^-(x) \end{pmatrix}_L \tag{26}$$

The upper components describe the 3 kinds of neutrino, while the bottom components describe the electron, muon and τ-lepton.

The right-handed components of leptons sit at $SU(2)$ singlets:

$$e^-_R(x),\ \mu^-_R(x),\ \tau^-_R(x) \tag{27}$$

They are right-handed Weyl spinors:

$$\gamma^5 e^-_R(x) = e^-_R(x) \tag{28}$$

Each left-handed dublet together with the corresponding right-handed singlet form a generation of leptons.

In what follows we concentrate on the first generation and introduce the notation

$$\begin{pmatrix} \nu_e(x) \\ e^-(x) \end{pmatrix}_L \equiv \begin{pmatrix} E^1_L(x) \\ E^2_L(x) \end{pmatrix},\ E_R(x) \equiv e^-_R(x) \tag{29}$$

Then the corresponding Lagrangian is given by

$$L_{lept} = \bar{E_L}^i(\imath\gamma^\nu D_\nu)E_L^i + \bar{E}_R(\imath\gamma^\nu D_\nu)E_R - \lambda_e\bar{E}_L^i\Phi^i E_R - \lambda_e\bar{E}_R(\Phi^i)^\dagger E_L^i,$$
$$D_\nu E_L^i = \partial_\nu E_L^i - \frac{\imath g}{2}A_\nu^a(\sigma^a)^i_j E_L^j - \frac{\imath \acute{g}}{2}B_\nu(Y_L)^i_j E_L^j,$$
$$D_\nu E_R = \partial_\nu E_R - \frac{\imath\acute{g}}{2}B_\nu Y_R E_R \tag{30}$$

where $i, j = 1, 2$ and λ_e is a coupling constant leading to the masses of leptons (λ_e is renormalizable constant so that it is a parameter of the model). Due to the vacuum averege (15) the leptons get masses

$$\Delta L_{lept} = -\lambda_e\bar{E}_L^i\Phi^i E_R - \lambda_e\bar{E}_R(\Phi^i)^\dagger E_L^i = -\frac{\lambda_e v}{\sqrt{2}}(\bar{e}_L e_R + \bar{e}_R e_L)... \Rightarrow$$
$$m_e = \frac{\lambda_e v}{\sqrt{2}} \tag{31}$$

Notice that we get the standard mass term for fermions $-\frac{\lambda_e v}{\sqrt{2}}\bar{e}e$ so that $e_L \to e_R$ transition is recovered. We could add the standard mass term for leptons directly but it would destroy the gauge invariance:

$$E_L(x) \to \exp(\imath\alpha^a(x)\frac{\sigma^a}{2} + \imath\beta(x)Y_L)E_L(x),$$
$$E_R(x) \to \exp(\imath\beta(x)Y_R)E_R(x) \tag{32}$$

Thus, **the Dirac's electron is a superposition of e_L and e_R which are completely different particles from the point of view of weak forces.**

To define Y_L and Y_R we use the relation (22). In the fundamental $SU(2)$-representation, which is used for the left-handed leptons

$$T^3 = \frac{1}{2}\sigma^3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \tag{33}$$

Hence,

$$Y_L = Q_{em} - T^3 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} \tag{34}$$

Right-handed leptons are singlets, that is $T^3 = 0$ and hence

$$Y_R = -1 \tag{35}$$

2.3. Quarks multiplets and Lagrangian.

Quarks are included into GWS model similarly to the leptons:

$$Q_1 = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}_L, \; Q_2 = \begin{pmatrix} c(x) \\ s(x) \end{pmatrix}_L, \; Q_3 = \begin{pmatrix} t(x) \\ b(x) \end{pmatrix}_L \tag{36}$$

Each component of the each dublet is a left-handed Weyl spinor w.r.t. Lorentz group.

The right-handed components of quarks sit at $SU(2)$ singlets:

$$u_R(x), \; d_R(x), \; c_R(x), \; s_R(x), \; t_R(x), \; b_R(x) \tag{37}$$

The qudruples (Q_1, u_R, d_R),...,(Q_3, t_R, b_R) form a generations of quarks.

The Lagrangian is given similar to (30). For the first quarks generation the Lagrangian is

$$\begin{aligned}
&L_q = \bar{Q_L}^i(\imath\gamma^\nu D_\nu)Q_L^i + \bar{u_R}(\imath\gamma^\nu D_\nu)u_R + \\
&\bar{d_R}(\imath\gamma^\nu D_\nu)d_R - \lambda_d \bar{Q}_L^i \Phi^i d_R - \lambda_d \bar{d}_R (\Phi^i)^\dagger Q_L^i \\
&-\lambda_u \epsilon^{ij} \bar{Q}_L^i \Phi^j u_R - \lambda_u \epsilon^{ij} \bar{u}_R (\Phi^j)^\dagger Q_L^i, \\
&D_\nu Q_L^i = \partial_\nu Q_L^i - \frac{\imath g}{2} A_\nu^a (\sigma^a)^i_j Q_L^j - \frac{\imath \acute{g}}{2} B_\nu (Y_L)^i_j Q_L^j, \\
&D_\nu u_R = \partial_\nu u_R - \frac{\imath \acute{g}}{2} B_\nu Y_{Ru} u_R, \\
&D_\nu d_R = \partial_\nu d_R - \frac{\imath \acute{g}}{2} B_\nu Y_{Rd} d_R
\end{aligned} \tag{38}$$

By the Higgs effect the quarks get the following masses:

$$
\begin{aligned}
&\Delta L_q = -\lambda_d \bar{Q}_L^i \Phi^i d_R - \lambda_d \bar{d}_R (\Phi^i)^\dagger Q_L^i \\
&-\lambda_u \epsilon^{ij} \bar{Q}_L^i \Phi^j u_R - \lambda_u \epsilon^{ij} \bar{u}_R (\Phi^j)^\dagger Q_L^i = \\
&-\frac{\lambda_d v}{\sqrt{2}}(\bar{d}_L d_R + \bar{d}_R d_L) - \frac{\lambda_u v}{\sqrt{2}}(\bar{u}_L u_R + \bar{u}_R u_L) ... \Rightarrow \\
&m_d = \frac{\lambda_d v}{\sqrt{2}},\ m_u = \frac{\lambda_u v}{\sqrt{2}}
\end{aligned} \tag{39}
$$

$\lambda_{u,d}$ are renormalizable constants so that they are the parameters of the model.

$$
Y_L = Q_{em} - T^3 = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} - \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{6} \end{pmatrix} \tag{40}
$$

The values of u, d quarks electric charges follow from the fact that proton has electric charge $+1$ and is a bound state of two u-quarks and one d-quark (in order to be colorless), while neutron has electric charge 0 and is a bound state of two d-quarks and one u-quark.

$$
p = \epsilon_{\alpha\beta\gamma} u^\alpha u^\beta d^\gamma,\ n = \epsilon_{\alpha\beta\gamma} d^\alpha d^\beta u^\gamma \tag{41}
$$

(Recall also the electric charge quantization phenomenon.)

For the right-handed quarks we find

$$
Y_R = Q_{em} = (\frac{2}{3}, -\frac{1}{3}) \tag{42}
$$

3. Standard Model.

3.1. Gauge group and bosonic sector.

The gauge group of the Standard model is

$$
SU(3) \times SU(2) \times U(1) \tag{43}
$$

where $SU(3)$ is responsible for the strong interractions of quarks and hence, we have to add strong interraction coupling constant g_s to the constants g, $\acute{g}$

of the electro-weak interraction. Thus the standard model have additional $SU(3)$-gauge symmetry and $SU(3)$ gauge fields transforming by the rule

$$
\begin{aligned}
&G^A_\mu(x)t^A \to U(x)G^A_\mu(x)t^AU^{-1}(x) + \frac{\imath}{g_s}U(x)\partial_\mu U^{-1}(x),\\
&U(x) = \exp\left(\imath\alpha^A(x)t^A\right) \in SU(3)
\end{aligned}
\tag{44}
$$

where t^A, $A = 1, ..., 8$ are generators of $su(3)$-Lie algebra

$$[t^A, t^B] = \imath f^{ABC}t^C \tag{45}$$

So we add to the electro-weak Lagragian the $SU(3)$ gauge fields contribution

$$
\begin{aligned}
&L_G = -\frac{1}{4}F^A_{\mu\nu}(F^A)^{\mu\nu},\\
&F^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f^{ABC}G^B_\mu G^C_\nu
\end{aligned}
\tag{46}
$$

The Higgs bosons are $SU(3)$-singlets so they do not interract to the $SU(3)$ gauge fields.

3.2. Quarks multiplets.

The quarks of all generations sit in the fundamental $SU(3)$-representation so that they are 3-components complex vectors regardless of chirality. In this representation the $su(3)$-generators are given by Gell-Mann matrices (see Appendix).

Thus, all the covariant derivatives from (38) have to be extended by $SU(3)$-gauge fields:

$$
\begin{aligned}
&D_\nu Q^{i\alpha}_L = \partial_\nu Q^{i\alpha}_L - \imath g_s G^A_\nu(t^A)^{\alpha\beta}Q^{j\beta}_L - \frac{\imath g}{2}A^a_\nu(\sigma^a)^i_j Q^{j\alpha}_L - \frac{\imath \acute{g}}{2}B_\nu(Y_L)^i_j Q^{j\alpha}_L,\\
&D_\nu u^\alpha_R = \partial_\nu u^\alpha_R - \imath g_s G^A_\nu(t^A)^{\alpha\beta}u^\beta_R - \frac{\imath \acute{g}}{2}B_\nu Y_{Ru}u^\alpha_R,\\
&D_\nu d^\alpha_R = \partial_\nu d^\alpha_R - \imath g_s G^A_\nu(t^A)^{\alpha\beta}d^\beta_R - \frac{\imath \acute{g}}{2}B_\nu Y_{Rd}d^\alpha_R
\end{aligned}
\tag{47}
$$

where $\alpha = 1, ..., 3$ labels the elements of $SU(3)$-multiplet. The quarks Lagrangian now takes the form

$$L_q = \bar{Q_L}^{i\alpha}(\imath\gamma^\nu D_\nu)^{\alpha\beta}Q_L^{i\beta} + \bar{u_R}^\alpha(\imath\gamma^\nu D_\nu)^{\alpha\beta}u_R^\beta +$$
$$\bar{d_R}^\alpha(\imath\gamma^\nu D_\nu)^{\alpha\beta}d_R^\beta - \lambda_d\bar{Q}_L^{i\alpha}\Phi^i d_R^\alpha - \lambda_d\bar{d}_R^\alpha(\Phi^i)^\dagger Q_L^{i\alpha}$$
$$-\lambda_u\epsilon^{ij}\bar{Q}_L^{i\alpha}\Phi^j u_R^\alpha - \lambda_u\epsilon^{ij}\bar{u}_R^\alpha(\Phi^j)^\dagger Q_L^{i\alpha} \quad (48)$$

Due to Higgs effect Yukawa interraction terms gives the standard mass terms for quarks

$$-\frac{1}{\sqrt{2}}\lambda_d v(\bar{d}_R d_L + \bar{d}_L d_R) - \frac{1}{\sqrt{2}}\lambda_u v(\bar{u}_R u_L + \bar{u}_L u_R) \quad (49)$$

3.3. Leptons multiplets.

The leptons of all generations sit in the $SU(3)$-singlets so they do not interract to $G_\mu^A(x)$ gauge fields.

Appendix.

Gell-Mann matrices:

$$t^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ t^2 = \begin{pmatrix} 0 & -\imath & 0 \\ \imath & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},\ t^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$t^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},\ t^5 = \begin{pmatrix} 0 & 0 & -\imath \\ 0 & 0 & 0 \\ \imath & 0 & 0 \end{pmatrix},\ t^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$
$$t^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\imath \\ 0 & \imath & 0 \end{pmatrix},\ t^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (50)$$